Dynamical Methods in Number Theory

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Here is the list of titles and abstracts for each lecture.

Lecture 1. The Birkhoff ergodic theorem. We will describe the fundamental concepts of ergodic theory and give an outline of a proof of the Birkhoff ergodic theorem.

Lecture 2. Markov shifts and the base-b map. A particularly simple example of an ergodic system is the base-b map, which encodes the digit distribution of the initial point in an orbit. We will prove the ergodicity (and more) of this map and continue with some arithmetic consequences; in particular an ergodic proof of mile Borel's classical result that almost all numbers are absolutely normal.

Lecture 3. Rotations of the circle and unique ergodicity. At the other extreme of the possible behaviours of circle maps from the Markov shifts of Lecture 2, we find rotations of the circle. These admit only a single preserved probability measure. We will give a detailed description of irrational rotations of the circle and show how the behaviour of orbits is completely governed by the arithmetic properties of the angle of rotation.

Lecture 4. The Gauss map and the Gauss measure. The Gauss map provides a dynamical description of the simple continued fraction algorithm. It has the nice feature of being ergodic with respect to a specific measure, the Gauss measure. We will outline a proof of this and deduce a number of classical results in the metric theory of continued fractions from the Birkhoff ergodic theorem.

Lecture 5. From continued fractions to homogeneous dynamics. In recent years, flows on homogeneous spaces have played a significant role in Diophantine approximation. We will give a description of the Gauss map in terms of the geodesic flow on a certain homogeneous space. Additionally, we will describe progress made in Diophantine approximation using this machinery.

I plan to slap up some lecture notes for the participants. Probably around 10 pages per lecture, so around 50 in total. I'll see if I can come up with some good exercises as well.