EXTERIOR AND SYMMETRIC ALGEBRA METHODS IN ALGEBRAIC COMBINATORICS

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1. LECTURE: GLICCI SIMPLICIAL COMPLEXES

One of the main open questions in liaison theory is whether every homogeneous Cohen-Macaulay ideal in a polynomial ring is glicci, i.e. if it is in the G-liaison class of a complete intersection; see [1] and [2] for details. An affirmative answer to this question can be given for Stanley-Reisner ideals defined by simplicial complexes that are weakly vertex-decomposable; see [3]. This class of complexes includes matroid, shifted and Gorenstein complexes respectively. Moreover, these constructions can be used to construct a simplicial complex which shows that the property of being glicci depends on the characteristic of the base field.

REFERENCES

[1] J. Kleppe, J. Migliore, R. M. Miró-Roig, U. Nagel, and C. Peterson, *Gorenstein liaison, complete intersection liaison invariants and unobstructedness*. Mem. Amer. Math. Soc. **154** (2001), no. 732.

[2] J. Migliore, *Introduction to Liaison Theory and Deficiency Modules*. Progress in Mathematics **165**, Birkhäuser 1998.

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2. LECTURE: H-VECTORS OF GORENSTEIN POLYTOPES

In this lecture we discuss results of [2]. We show that the Ehrhart h-vector of an integer Gorenstein polytope P with a regular unimodular triangulation satisfies McMullen's g-theorem; in particular, it is unimodal. This result generalizes a theorem of Athanasiadis [1] (conjectured by Stanley; see [3]) for compressed polytopes. It is derived from a more general theorem on Gorenstein affine normal monoids M: one can factor $K[M]$ (K a field) by a "long" regular sequence in such a way that the quotient is still a normal affine monoid algebra. This technique reduces all questions about the Ehrhart h-vector of P to the Ehrhart h-vector of a Gorenstein polytope Q with exactly one interior lattice point, provided each lattice point in a multiple cP, $c \in N$, can be written as the sum of n lattice points in P. If P has a regular unimodular triangulation, then it follows readily that the

Ehrhart h-vector of P coincides with the combinatorial h-vector of the boundary complex of a simplicial polytope, and the g-theorem applies.

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3. LECTURE: KOSZUL HOMOLOGY AND SYZYGIES OF VERONESE SUBALGEBRAS

Let R be a finitely generated graded K-algebra where K is a field. We say that R has property N_p if it is generated in degree 1, has relations in degree 2 and the syzygies of order $\leq p$ on the relations are linear; see [2, 3] for details. The Green-Lazarsfeld index of R is the largest p such that it satisfies the property N_p. In this lecture we discuss results of Bruns, Conca and myself related to this index; see [1]. The main theorem is that (under a mild assumption on the base field) the c-th Veronese subring of a polynomial ring has Green-Lazarsfeld index greater or equal to $c+1$. The same conclusion also holds for an arbitrary standard graded algebra, provided c is sufficiently large.

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4. LECTURE: HOMOLOGICAL PROPERTIES OF ORLIK-SOLOMON ALGEBRAS

The Orlik-Solomon algebra of a matroid can be considered as a quotient ring over the exterior algebra E; see [2] and [3] for details. In this lecture we present recent results of Kämpf and Römer [1]. At first we study homological properties of E-modules as e.g. complexity, depth and regularity. In particular, we consider modules with linear injective resolutions. We apply our results to Orlik-Solomon algebras of matroids and give formulas for the complexity, depth and regularity of such rings in terms of invariants of the matroid. Moreover, we characterize those matroids whose Orlik-Solomon ideal has a linear projective resolution and compute in these cases the Betti numbers of the ideal.

REFERENCES

[1] G. Kämpf and T. Römer, *Homological properties of Orlik-Solomon algebras*. To appear in Manuscr. Math.

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