

COMBINATORICS OF TRIANGULATIONS AND HILBERT-SERIES

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Lahore, February 21–28, 2009

1. LECTURE: REVIEW OF GRÖBNER BASES AND BASICS OF POLYTOPE THEORY

We introduce the basic concepts of Gröbner basis theory and its relations to polytope theory. This will cover very basic parts of [1], the basic chapters from [3] and the first chapters from [11]. In particular, we will define the Gröbner fan, which will play a major role in some of the research problems we will pose later.

2. LECTURE: TRIANGULATIONS, GRÖBNER BASES, f -VECTORS AND HILBERT-SERIES

In this lecture we relate triangulations to Gröbner bases and Hilbert-series. In particular, we study f -vectors of simplicial complexes and its Stanley-Reisner ideals. The basic material can be found in the book [4] and the book [11].

3. LECTURE: GRÖBNER BASES FOR HIBI-RINGS AND OTHER EHRHART-RINGS

In this lecture we exhibit the connections between the Neggers-Stanley conjecture, Hibi-rings and Gröbner basis. Then we present the results that show how to generalize this to arbitrary Ehrhart-rings. The lecture is based on results from [7], [2] and [5].

4. LECTURE: SPECIAL GRÖBNER BASES FOR DETERMINANTAL AND PFAFFIAN IDEALS

In this lecture we present special Gröbner bases for Determinantal and Pfaffian ideals that allow strong conclusions on the Hilbert-series. In particular, we relate those Gröbner bases to polytope theory and classical polytopes such as the associahedron and the cyclohedron. This lecture is based on [6], [10] and background from [9] and [8].

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