

Computing in Commutative Algebra

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1. Lecture: Gröbner bases and SINGULAR

The notion of a Gröbner basis (with respect to any ordering) will be explained as the basis for computations in localizations of factorings of polynomial rings. The computer algebra system SINGULAR and its use will be explained as a basis for further applications in commutative algebra and other fields.

2. Lecture: Polynomial solving and primary decomposition

Triangular sets as a symbolic pre-processing to solve polynomial systems of equations will be discussed. The computation of a primary decomposition - one of the most difficult task in computer algebra - will be explained. The computation of the normalization of a ring will be included.

3. Lecture: Invariants

The computation of many invariants in commutative algebra as Hilbert function, Hilbert polynomial, dimension, degree, multiplicity will be discussed and explained.

4. Lecture: Homological Algebra

Important notions in homological algebra as for instance depth and Cohen-Macaulay-ness will be analyzed and it will be explained how to check respectively compute it.

References

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Computer Algebra Systems

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- CoCoA (Robbiano, L.): A System for Computation in Algebraic Geometry and Commutative Algebra.
Available from cocoa.dima.unige.it/cocoa
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- SINGULAR (Greuel, G.-M.; Pfister, G.; Schönemann, H.): A Computer Algebra System for Polynomial Computations. Centre for Computer Algebra, University of Kaiserslautern, free software under the GNU General Public Licence (1990-2007).
<http://www.singular.uni-kl.de>.