 Max Time: 90 min

Student

3-Point-Problems

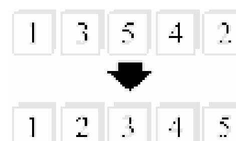
1. For which of the following values of x is the value of the expression $\frac{x^2}{x^3}$ the smallest?

(A) 2 (B) 1 (C) −1 (D) −2 (E) −3

2. How many numbers from 2 to 100 are equal to the cube of an integer?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

3. Five cards numbered from 1 to 5 are arranged as shown.
Per move, any two cards may be interchanged.
Find the smallest number of moves required to arrange them in increasing order.

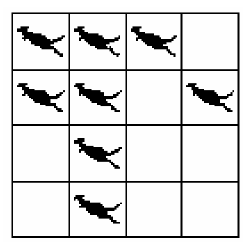


(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4. If $888 \cdot 111 = 2 \cdot (2 \cdot n)^2$, and n is a positive integer, n equals

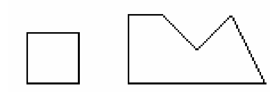
(A) 8 (B) 11 (C) 22 (D) 111 (E) 444

5. There are eight kangaroos in the squares of the table as shown.
A kangaroo can move from its square directly into any empty square.
Find the least number of moves so that exactly two kangaroos remain In any row and in any column of the table.

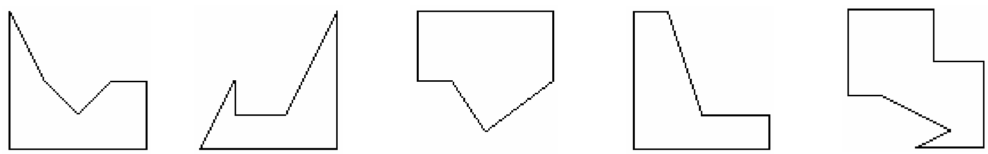


(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

6. A square piece of paper has been cut into 3 pieces. Two of them are



What is the shape of the third piece?

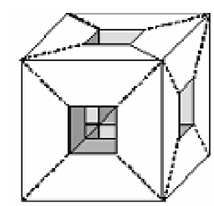


(A) (B) (C) (D) (E)

7. The sum of four consecutive positive integers cannot be equal to

(A) 2002 (B) 22 (C) 202 (D) 222 (E) 220

8. A $3\times3\times3$ cube weighs 810 grams. If we drill three holes through it as shown, each of which is a $1\times1\times3$ rectangular parallelepiped, the weight of the remaining solid is

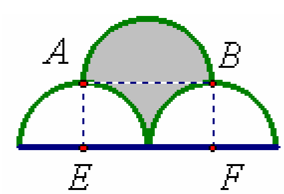


(A) 540 g (B) 570 g (C) 600 g (D) 630 g (E) 660 g

9. If f is a function such that $f(x+1) = 2f(x) - 2002$ holds for all integer values of x and $f(2005) = 2008$, then $f(2004)$ equals:

(A) 2004 (B) 2005 (C) 2008 (D) 2010 (E) 2016

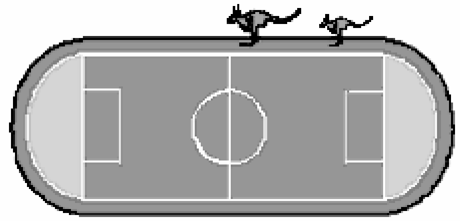
10. We are given three semi-circles as shown. *ABEF* is a rectangle and the radius of each of the semi-circles is 2 cm. *E* and *F* are the centers of the bottom semi-circles. The area of the shaded region in cm^2 is



(A) 8 (B) 7 (C) 2π (D) $2\pi + 1$ (E) $2\pi + 2$

4-Point-Problems

11. Mom Kangaroo and her child Jumpy are jumping around a stadium with perimeter 330 m. They each make one leap per second. Mom leaps 5 m each time, and Jumpy 2 m. They both start at the same point and jump in the same direction.



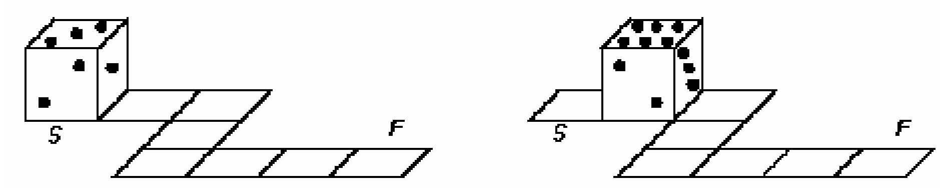
After 25 seconds Jumpy gets tired and stops while his mother continues. How long does it take her to overtake Jumpy?

(A) 15 sec (B) 24 sec (C) 40 sec (D) 51 sec (E) 66 sec

12. Henny paints each face of several wooden cubes white or black, using both colors on each cube. How many different colorings are possible?

(A) 8 (B) 16 (C) 32 (D) 52 (E) 64

13. The sum of dots on opposite faces of a die always equals 7. A die rolls as shown below.



At the starting point (*S*) the top face is 3. Which will be the face at the end point (*F*)?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

14. A box contains 60 tickets: some red, some blue and some white. If all red tickets were replaced by blue tickets, then there would be twice as many blue tickets as white tickets; but if all the white tickets were replaced with blue ones, then there would be three times as many blue tickets as red tickets. The number of blue tickets in the box is

(A) 10 (B) 15 (C) 20 (D) 25 (E) 30

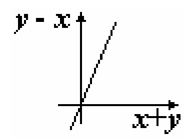
15. Let a and b be the lengths of the sides of the right-angled triangle. If d is the diameter of the incircle and D is the diameter of the circumcircle of this triangle, then $d + D$ is equal to

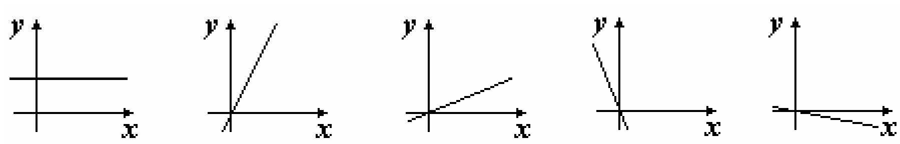
(A) $a+b$ (B) $2\cdot(a+b)$ (C) $\frac{1}{2}\cdot(a+b)$ (D) $\sqrt{a\cdot b}$ (E) $\sqrt{a^2+b^2}$

16. Let M be the set of all real numbers x for which the inequality $2^{4^x} < 4^{2^x}$ holds. Then $M =$

(A) $(-\infty, 1)$ (B) $(0,1)$ (C) $(-\infty, 1) \cap (1, \infty)$ (D) $(0, \infty)$ (E)

17. On the picture you can see the graph for relation between $y - x$ and $y + x$. Find the graph for relation between x and y.



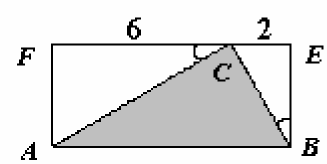


(A) (B) (C) (D) (E)

18. Two bottles of equal volume are both filled with a solution of water and acid. The ratios of the volume of water to acid are, respectively, 2 : 1 and 4 : 1. We pour all the contents of the two bottles into one big bottle. Then the ratio of water to acid in this bottle will be:

(A) 3 : 1 (B) 6 : 1 (C) 11 : 4 (D) 5 : 1 (E) 8 : 1

19. The diagram shows a rectangle *ABEF* and a triangle *ABC*. We know that the angle *ACF* equals angle *CBE*. If $FC = 6$ and $CE = 2$ then the area of *ABC* is:



(A) 12 (B) 16 (C) $8\sqrt{2}$ (D) $8\sqrt{3}$ (E) non of these

20. Sherlock Holmes and Dr. Watson went from London in Plymouth. When they go out, Dr. Watson has asked: "Holmes, how long a time we were in a train?". "I do not know, - has answered Holmes, - but I have noticed, that at the moment of departure, and now, when we have arrived, the angle between hour and minute arrows of my watch was a 90°". If distance from London up to Plymouth of 120 km, what can be a speed of a train?

(A) 120 km/h (B) 110 km/h (C) 100 km/h
(D) 60 km/h (E) any of listed

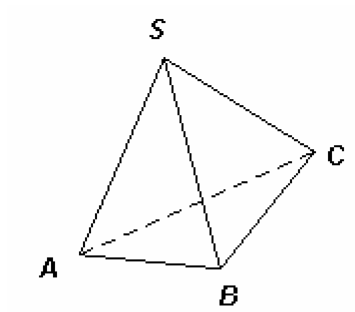
5-Point-Problems

21. Which of the following numbers can be expressed as the product of four different integers, each of them greater than 1?

(A) 625 (B) 124 (C) 108 (D) 2187 (E) 2025

22. In the pyramid *SABC* all plane angles with vertex *S* are equal to 90°. The areas of the lateral faces *SAB*, *SAC* and *SBC* are 3, 4 and 6, respectively. Find the volume of *SABC*.

(A) 4 (B) 5 (C) 6 (D) 8 (E) 12



23. If the sum of the digits of a number m is 30, then the sum of the digits of the number $m+3$ cannot be

(A) 6 (B) 15 (C) 21 (D) 24 (E) 33

24. In a bag we have 17 balls numbered by $5 + k \cdot 125$, $k = 0, \ldots, 16$, i.e. by 5, 130, 255, 380, 505, . . . 1755, 1880, 2005. If we select several balls at random, what is the smallest number of balls needed to guarantee that the selection contains at least one pair of balls that add up to 2010?

(A) 7 (B) 8 (C) 10 (D) 11 (E) 17

25. If we know that $\log_{10}(\sqrt{2005} + \sqrt{1995}) = n$ which of the following is the value of $\log_{10}(\sqrt{2005} - \sqrt{1995})$?

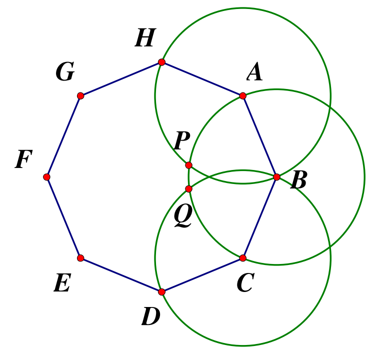
(A) $n - 1$ (B) $1 - n$ (C) $\frac{1}{n}$ (D) $n + 1$

(E) Impossible to determine with the information given

26. The integer A has exactly two divisors. The integer B has exactly five divisors. How many divisors does the number $A \cdot B$ have?

(A) 5 (B) 6 (C) 7 (D) 10
(E) It is not possible to determine without additional information.

27. In the figure, *ABCDEFGH* is a regular octagon of side 1. Points *P* and *Q* are the intersections of circles with centers in *A*, *B* and *C* and radius 1. What is the size of $\angle APQ$?



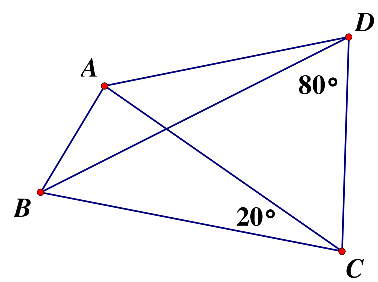
(A) $\frac{19}{24}\pi$ (B) $\frac{8}{11}\pi$ (C) $\frac{5}{8}\pi$

(D) $\frac{3}{4}\pi$ (E) $\frac{7}{9}\pi$

28. Start with a number, double it and then subtract 1. After applying this procedure 98 more times (starting each time from the previous result) you get $2^{100}+1$. Which was the number you started with?

(A) 1 (B) 2 (C) 4 (D) 6 (E) none of these

29. In the quadrilateral *ABCD* the diagonal *BD* is the bisector of $\angle ABC$ and $AC = BC$. Given $\angle BDC = 80°$ and $\angle ACB = 20°$, $\angle BAD$ is equal to



(A) 90° (B) 100° (C) 110°
(D) 120° (E) 135°

30. Henry must travel from *A* to *B* and he plans to go at a certain speed. He would like to arrive earlier than planned and notes that traveling at a speed 5 km/h faster than planned he will arrive 5 hours earlier and traveling at a speed 10 km/h faster than planned he will arrive 8 hours earlier. His planned speed is

(A) 10 km/h (B) 15 km/h (C) 20 km/h
(D) 25 km/h (E) impossible to determine