

BASIC COURSE FOR IMO TRAINING CAMP

ALGEBRA

Polynomials:

Synthetic division, Remainder theorem, Factorization, Symmetric, alternating and cyclic functions.

Complex Numbers:

Fundamental operations, Argand diagram, DeMoivre's theorem, Roots of complex number.

Theory of Equations:

Roots of a polynomial equation, Relations connecting the roots and coefficients, Transformation of equations, Character and position of roots, Descartes rule of signs, Limits to the roots, Finding rational roots, Newton's method of divisors, Symmetric functions of roots.

Inequalities:

Arithmetic and geometric mean (AM-GM) inequality, Weighted AM-GM inequality, Cauchy's-Schwarz inequality, Holder's inequality, Minkowski's inequality, Bernoulli's inequality.

Reference Books:

1. *Higher Algebra* by S. Barnard
2. *Higher Algebra* by Hall H. S

COMBINATORICS

Basic Counting Techniques:

The addition rule, The multiplication rule, Possibility tree, Permutations, Permutations with repetition, Combinations, Combinations with repetition, Binomial theorem, Algebraic and combinatorial proofs, Permutations with indistinguishable objects.

Further Topics in Enumeration:

The pigeonhole principle, The principle of inclusion and exclusion, Mathematical Induction, Recurrence relations, Classical problems like Towers of Hanoi, The Fibonacci numbers, Number of partitions of a set into r subsets etc., Solving recurrence relations by iteration.

Graph Theory & Applications:

Introduction to Graphs, Paths and circuits, Connectedness, Euler path and circuit, Graph isomorphism, Planer Graphs, Graph coloring, Introduction to trees, Rooted trees, Binary trees, Tree traversal.

Reference Books:

1. *Discrete Mathematics and its Applications*, 5th edition, by Kenneth H. Rosen
2. *Discrete Mathematics with Applications*, 2th edition, by Susanna S. Epp
3. *Discrete mathematics with Graph Theory*, 2nd edition, by Edgar G. Goodaire, Michael M. Parmenter

GEOMETRY

Triangles and Trigonometry:

Elements of a triangle, Triangles classified according to sides/angles, Medians, altitudes, angle bisectors and right bisectors (centroid, orthocenter, in center and circum center) of a triangle.

Congruent triangles, Basic principles of congruent triangles (SAS, SSS, ASA), Principles of proportional segments, Similar triangles, Principles of similar triangles.

Circles:

Basic terminology, Circle principles, Tangent to a circle, Tangent principles, Measurement of angles and arcs in a circle, Inscribed circle, Circumscribed circle, Escribed circle, Nine point circle and Euler line.

Areas:

Area of a rectangle, square, parallelogram, triangle, trapezoid, rhombus. Comparing areas of similar polygons.

Regular Polygons and the Circle:

Polygons, Inscribed and circumscribed polygons, Regular polygons, Regular polygon principles, Area of a regular polygon, Ratios of segments and areas of regular polygons, Area of a sector and a segment.

Geometric Inequalities:

Triangle inequalities, Circle inequalities, Geometrical, trigonometrical and algebraic presentation of the same inequality,

Reference Books:

1. *Geometry, Schaum's Outline series, 3rd ed.*, by Barnett Rich
2. *Plane Euclidean Geometry: Theory and Problems*, by Geoff Smith

NUMBER THEORY

Divisibility Theory:

The division algorithm, Divisibility of integers, Prime numbers, Fundamental theorem of arithmetic, Greatest common divisor (gcd), Euclidean algorithm, Least common multiple, The number and sum of divisors

Congruences:

Modular arithmetic, Linear congruences, Chinese remainder theorem, Wilson's theorem, Euler's criteria, Fermat's little theorem, Euler's ϕ function and Euler's theorem

Quadratic Congruences:

General quadratic congruence, Quadratic residues, Gauss's law of quadratic reciprocity

Diophantine equations:

Linear equations, Pythagorean triples, Fermat's last theorem, Sum of two squares, Sum of four squares, Sum of three squares

Reference Books:

1. *The Theory of Numbers* by Andrew Adler
2. *Elementary Number Theory with Applications* by Thomas Koshy
3. *Elementary Number Theory* by David M. Burton