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RAISING THE PROFILE OF MATHEMATICS

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1. Introduction

Over the half century or so in which I have been exposed to maths, I have come to the conclusion that the subject is not given the importance it deserves. I want to explicate this vague statement, and suggest how one might begin to improve matters. There is nothing new here - this talk is a selection of established facts and informed views, chosen to give coherence to my feelings. But this seems to be a "pregnant" issue, on which it might be helpful to catalyse debate. I am grateful to Dr.A.D.R.Choudary[1], who kindly invited me to visit Lahore, and whose idea it was to talk on this subject.

I need to begin with a stark reality. In the present global economy, prosperity depends on scientific and engineering strength, which in turn is built on the foundation of mathematics. That is the reason why mathematics is of prime importance in any developed country. The world's most successful economies all have historical strength in mathematics. It is, quite simply, one of the keys to the future. And yet ...

2. Not All Is Well

Despite its intrinsic importance, not all is well with maths. Here is a quote from a Royal Society of London report, just five years ago:

In launching UK Maths Year 2000, the Prime Minister made clear the importance of mathematics in the education of those creative and flexible thinkers

[1]Director-General of School of Mathematical Sciences, GC University, Lahore, Pakistan

on whom our national economic prosperity will depend. The demands of commerce and industry for articulate graduates with mathematical skills far outstrips the supply of graduates in mathematics and related fields.

In fact the problem is a global one, faced by all developed nations. Maths has too low a billing in national education programs. Far too few students study the subject, especially at the levels required by cutting edge scientific and technological research. Several governments have expressed concern, resulting in much discussion. It is not enough simply to recognise that there is a problem – we must begin to understand that problem. It is increasingly clear that too many policymakers understand neither the intrinsic importance of mathematics, nor the problems facing it. There are plenty of good, committed mathematicians who (given the chance) are capable of rectifying the situation. Sadly, all too often they receive neither the necessary support, nor the direction. How has this situation arisen?

3. **Myths and Realities**

In 1991 the US National Research Council issued a report on mathematics education in which (amongst much else) it refutes a number of myths on the basis of serious research. I want to quote some of the myths, and the realities.

- *Success in maths depends more on innate ability than on hard work.* Sustained effort can carry most students to a satisfactory level of achievement. When parents or teachers believe that genetic ability is the primary factor contributing to success, students are likely to fail before they begin: when expectations are high, so is the resulting performance.
- *Most jobs require little maths.* Far from it. Although arithmetic sufficed in the past, ever more jobs (especially those using computers) require developed quantitative skills.
- *All useful maths was discovered long ago.* Discoveries in maths are essential for industrial competitiveness. Without them we would not have modern communications, computers, aircraft, satellites or equity markets. More new maths is being created and used now than ever before in history.
- *Maths is just calculating answers.* Real problems are often ambiguous, admitting many forms and several answers. Mathematical power is revealed as much by the act of identifying and properly posing problems as by the application of specific techniques and algorithms.
- *Only scientists and engineers need to study maths.* Mathematics is a science of patterns that applies to all disciplines. The most rapid areas

of growth have been in the social, biological, behavioural, communication, demographic, financial and legal fields.

As the report points out, these pernicious myths spread ignorance like a cancer, and excuse underachievement throughout society. They can only be eradicated by sustained effort at all educational levels, especially in colleges and universities where the nation's leaders are educated. Part of the problem is that *the role of maths in its applications is largely hidden.* Think of the use in the security industry of biometric information, such as fingerprints and iris scans. What you see is a result on a computer screen within a fraction of a second. But the technology arose from extensive mathematical research. A mathematical result has given rise to an algorithm, which has been programmed by computer scientists, and then embedded in a chip. Industries buy electronic hardware. They do not see, nor need to be aware of, the underlying maths. Thus the perception of technology endusers is that the hardware is all–important to them the maths is invisible.

4. **The Shape of Mathematics**

In his excellent paper on "Mathematical Education", the American mathematician W.Thurston introduced some useful terms, which help to phrase some of the problems faced by the subject. He points out that mathematics has a "shape". We tend to think of maths as a pyramid: however, it is more like scaffolding, with many interconnected supports. Whenever a given level of the scaffolding is solidly in place, it is not hard to build it higher, but it is impossible to build a layer before previous layers are in place. Because reasoning in maths is so certain, it is possible to build structures which are very tall, very reliable, and very powerful. Mathematics is both a very "tall" structure (new concepts may depend on a large number of previous concepts) and a very "wide" subject (there are numerous areas of the subject for which even the beginnings do not appear in undergraduate syllabuses).

There appear to be two problems at the school level. One is that over the last few decades the syllabuses have become increasingly *narrow*, concentrating on a small number of topics to the exclusion of many others. The other is that they have become increasingly *tall*, with material introduced from increasingly higher maths levels. The result is that many students have a maths education which is narrow and tall. It reaches a certain height, above which its base supports no more growth, and there it halts. Thurston's view is that this trend has been exacerbated by an over-reliance on standardised tests. The way forward is clear. We need to decrease the height of the school maths curriculum, and increase the width. To decrease the height we will have to learn to be content with attaining less in some directions: and to increase the width we need to think seriously about introducing more maths topics at elementary levels.

Of course, from the purely educational point of view, there is much to be said for the greatest breadth possible, linking maths as much as possible to other subjects, and thereby making more sense of the subject for the students. And by reducing the height one allow students to gain a much firmer understanding of the fundamentals, gaining real fluency in elementary techniques, and thereby gaining confidence. Although all this has been phrased for *school* curricula, my personal feeling is that it applies equally to *university* studies. We should not aim to satisfy just the tiny minority of students with evident mathematical ability, but rather the majority who require a good understanding of the basics to fulfill their obligations in the workplace. Able students will be warmly welcomed at higher educational levels, where their needs will be met.

5. **The Role of Geometry**

Perhaps the most obvious casualty of the trend to narrow curricula is elementary geometry. That brings me to a matter close to my own heart, namely the role of geometry within the subject. The first point I want to make is that geometry is intrinsically important *within maths itself*. Why is this so? Let me quote a fascinating observation of Sir Michael Atiyah, an outstanding contributor to world class mathematics:

Our brains have been constructed in such a way that they are extremely concerned with vision. ... Vision, I understand from friends who work in neurophysiology, uses up something like 80 or 90 percent of the cortex of the brain... Understanding, and making sense of, the world that we see is a very important part of our evolution. Therefore spatial intuition or spatial perception is an enormously powerful tool and that is why geometry is actually such a powerful part of mathematics - not only for things that are obviously geometrical, but even for things that are not. We try to put them into geometrical form because that enables us to use our intuition. Our intuition is our most powerful tool ... I think it is very fundamental that the human mind has evolved with this enormous capacity to absorb a vast amount of information, by instantaneous visual action, and mathematics takes that and perfects it.

Geometry is also central to modern mathematical applications. It contributes significantly to communications, computer vision, demography, finance, genetics, geophysical and ionospheric prediction, weather forecasting, robotics, speech recognition, tomography and a host of other fields. Moreover there is a psychological aspect. Few would deny that substantial areas of maths fail to excite student interest: yet there are many students attracted to geometry by its sheer visual content. Numerous distinguished academics and organisations have concluded that geometry must regain its rightful place in mathematics teaching. The Royal Society report made the following recommendation to Government:

It is a matter of national importance that as many of our students as possible fully develop their mathematical potential. Geometry, with its distinctive appeal, should make mathematics attractive to a wider range of students. ... For some students it may be the logical aspects which are the most appealing, for others it may be the visualising, or the modelling, or the historical and cultural, or the visual and aesthetic aspects.

There is a wealth of different approaches here. My preference is to focus on plane curves. They live in an environment familiar from school maths (the Euclidean plane) and have features readily visible on a computer screen: indeed, students can have fun tracing curves using programs such as MAPLE. Moreover, their study uses foundational maths (calculus, linear algebra, complex numbers) in a useful way. Most calculations use no more than elementary symbolic manipulation and differentiation, reinforcing basic mathematical skills. Lines and circles should be familiar from school, whilst conics are fundamental in several areas. That leads naturally to a zoo of classical curves, and in turn to the beginnings of algebraic and differential geometry, which play an important role in geometric applications. However, there are other approaches to the introduction of geometry, such as the interesting set of notes "Geometry and the Imagination" due to Conway, Doyle and Thurston[2].

6. **Some of the Needs**

In order to serve society, we need to raise the numbers of people studying maths at higher levels, and ensure that they are properly trained in relevant areas. How can we approach this problem? Let's split the big problem into a number of smaller problems, each better defined, and more tractable.

- *Communication within maths needs to improve.* The whole maths community is pigeonholed by level, specialism, and geography. These barriers prevent interaction, and hinder development. For instance, university staff and school teachers can learn from each other: too many professors know little about schools, and too many teachers feel insecure in their knowledge, so are timid about approaching anyone.

[2]These notes can be downloaded from the University of Minnesota Geometry Center.

To break down these barriers we need vehicles for the exchange of interesting educational ideas. Increasingly, websites and internet forums offer realistic solutions to this problem.

- *We need to make maths more attractive.* Too many students are unresponsive, afraid of making guesses. We need to foster their natural interest and curiosity. Raising the profile of maths will help maths become more attractive to young people. So will the introduction of geometry.
- *People learn maths by doing it.* Although known to be ineffective, large class lecturing dominates maths teaching, because it is cheap. Interactive tutorials, computer aided learning, problem classes and projects represent interesting ways forward.
- *We need to broaden the university base.* Curricula tend to be too tall, and too narrow. We need to introduce more basic topics, linking maths to other subjects, and not be driven by the desire simply to attain distance in some direction.
- *Maths needs to involve computing.* Computers have significantly changed the practice of maths, at all levels. They enhance student motivation, provide natural catalysts for teamwork, and focus attention on learning.
- *The system of professional rewards needs changing.* We should recognise that different people can make significant contributions in different ways. Innovative teaching, high quality research, and imaginative leadership all need to be properly rewarded.

7. **Raising the Profile**

I want to outline a barebones strategy for raising the national profile of mathematics. That will automatically raise the profile of science and technology, which can only benefit the economy. I think the way forward is a combination of the "top–down" and "bottom–up" approaches.

By the "top–down" approach I mean that government needs to be convinced of the intrinsic importance of maths to the long term health of the economy, and to institute a program ensuring greater support at all levels for the subject. Clear support from government would certainly stimulate change, and encourage more people to study maths. I am sure that experienced mathematicians with proven track records would do their very best to offer government the best possible advice. A starting point is to ensure that maths is well taught in all schools, by fully qualified teachers. Gifted pupils can be encouraged to continue their studies by a system of government scholarships. At the university level, maths needs to be properly funded and supported. Strong research schools need to be set up, with a clear mandate to provide the skills necessary for developments in science and technology. Government funding (possibly in tandem with industrial funding) should be available on a competitive basis for the highest quality projects.

Simultaneously one needs the "bottom–up" approach in which the mathematical community itself takes steps to make maths a more attractive subject to study, and to ensure that those facets of the subject directly relevant to the needs of society are taught. As I explained above, there is much that can be done. I think university departments need to take positive steps to involve themselves in maths at the school level. The introduction of geometry would be a major improvement, as would a commitment to link maths more systematically to other subjects, in particular computer programming.

There is considerable room in these approaches for TV advertisement. I see every good reason for furthering the interests of academic subjects by providing sensible media coverage, aimed at the general public. Competitive logic games, such as chess and SuDoKu, have considerable appeal. Likewise, participation in international Mathematical Olympiads is one way of picking out high fliers. TV programme makers should be encouraged to consider making documentaries relating to mathematical concepts. For instance the key concept of symmetry plays a significant role in classical architecture, and there is cultural value in bringing these visual and aesthetic aspects to public attention.

8. **The Gentle Revolution in Pakistan**

What of the future in Pakistan?

This revolution has already started at the primary and high school level through the recent introduction of the International Kangaroo Program, and the Mathematical Olympiad. Pakistani youth are already being stimulated to take an interest in a key future subject. Undoubtedly, more can be done. Maths Roadshows have already had considerable success in Europe and the US, and could be introduced to Pakistan both cheaply and easily. Websites and Internet Forums will also provide ways of increasing communication within the Mathematical Community.

At the higher education level, the biggest single change will be the introduction of a 4-year BS degree, enabling Pakistan to compete with developed nations in this field. Moreover, the content of the degree will undergo substantial change, to ensure that Pakistani graduates are properly trained in the areas of mathematics supporting cutting edge technological research. Beyond this, discussions are under way to raise the standard of mathematics research by altering the structure of the MS and PhD degrees. Together with quality control of teaching and research these measure will place Pakistan in a very strong future position.

Pakistan is one of the five largest nations on earth. Given goodwill by Government and the Academic Community, there is no reason why it should not boast one of the best universities in the world within our lifetimes. And that will represent one part of the transmogrification that is taking place in one of the most significant regions on earth.