

Boundedness Criteria for Maximal Functions and Potentials on the Half-space in Weighted Lebesgue Spaces with Variable Exponents

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ABSTRACT

Necessary and sufficient conditions on weight pair (v, w) guaranteeing the boundedness of generalized maximal functions and potentials on the half-space from $L_w^p(\mathbb{R}^n)$ to $L_v^{q(\cdot)}(\mathbb{R}_+^{n+1})$ are found, provided that p is a constant and $w^{-p'}$ satisfies the dyadic reverse doubling condition. Carleson-type criteria governing the trace inequality for generalized fractional maximal functions and potentials on the half-space are also derived. For fractional maximal functions we use the technique of dyadic cubes and Carleson-Hörmander inequality, while the proofs for fractional integrals rely on the Welland-type inequality.

In the diagonal case our conditions are of Sawyer type. The results are new even for constant exponents of Lebesgue spaces.