Black Hole Paradoxes

Asghar Qadir

Department of Physics

School of Natural Sciences (SoNS)

National University of Sciences & Technology (NUST)

Islamabad, PAKISTAN

NCM & AS-SMS Lahore 26 March 2018

Introduction

- When Stephen Hawking put his PhD thesis in the open domain on the net some 100,000 copies were downloaded, causing the site to crash repeatedly. There are certainly not 10,000, and perhaps not even 1,000, who can follow the Mathematics in that thesis.

- One concludes that the downloads were obtained because Hawking has become a cult figure and not because people expected to follow what he had said. This is borne out by the comments of people who had downloaded and wrote on a blog.

Introduction

- When Susskind and Hawking had a bet about Hawking's "information loss paradox" in 1997, it was noted only by some of the physicists interested in black holes.
- When Hawking paid for losing the bet in 2004 — with claims of not being *actually* wrong, but admitting that he *had* lost the bet — it became very big news.
- Partly, it was because he had become more of a cult figure by then, and partly because

  "Hawking was *actually* wrong!"

Introduction

- When I came across Hawking's work on radiation from black holes in 1976, I was excited at this attempt to provide a "halfway house" for "the quantization of gravity", as it is called.
- However, I found that some of the ideas did not seem to lead to reasonable results — they were paradoxical.

Introduction

- The work I did when I visited John Wheeler in Texas in 1978 led to some questions about an ambiguity in what were then called “Penrose diagrams”, whose resolution gives some insights into the spacetime structure of black holes.
- I found, in the early years of this millennium that the history of the introduction of thermodynamics into black hole discussions, was being distorted, so that even its author had forgotten his contribution and needed to be reminded of it.

Introduction

- Again in the early years of this millennium, I found that there is a problem with the claims of computing the entropy of 2-d black holes.
- My work with Wheeler, in 1986, led to some conclusions that suggested that black holes need not be “the end of the matter”, and suggest a nearly classical resolution to the information loss paradox, which should give a different picture to the Susskind-’t Hooft resolution.

Introduction

- In the first three talks I will discuss these issues and in the fourth, I will present a proposed experiment to test between different, so-called, interpretations of Quantum Theory. The plan of the talks is as follows:

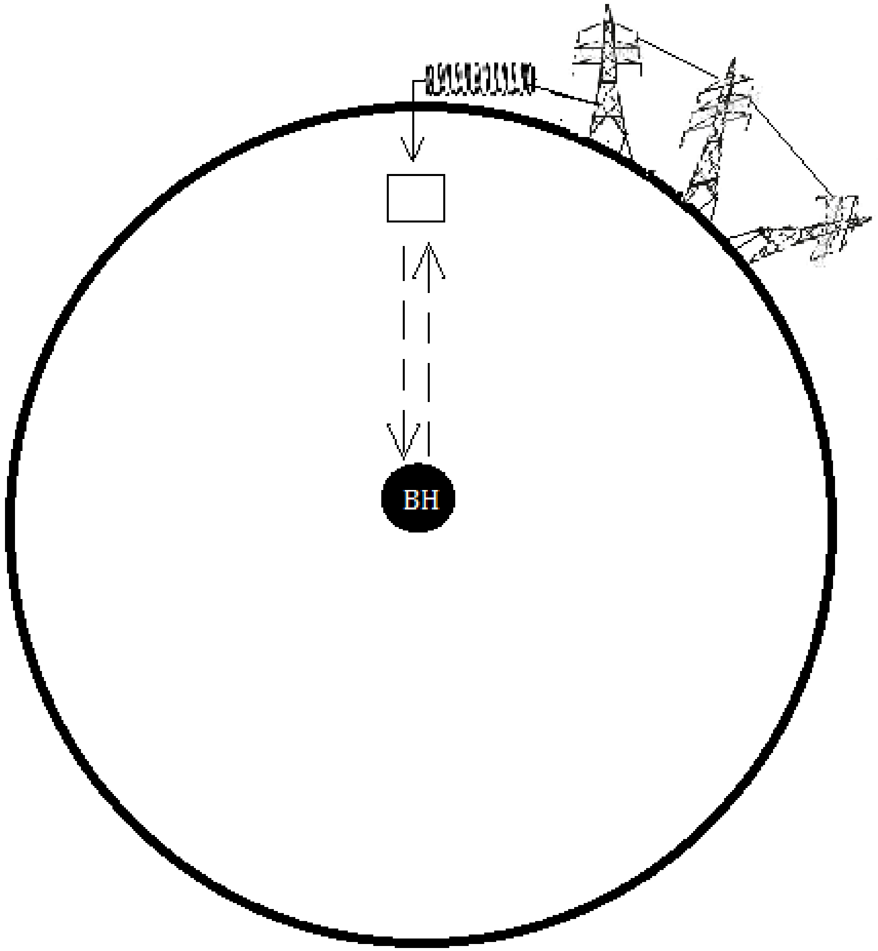
1. The chronology of black hole thermodynamics;
2. My black hole paradoxes;
3. The ambiguity in the Carter-Penrose diagram;
4. A nearly classical resolution of the information loss paradox;
5. The problem with 2-*d* black holes;
6. Conclusion.
7. Proposed experiment to test “interpretations” of Quantum Theory.

Chronology of Black Hole Thermodynamics

- When I joined the PhD at Birkbeck College, London University in 1968, Roger Penrose used to hold lunch time seminars. At one of these he presented an interesting idea.
- He addressed the question whether black holes could not be used to violate the second law of thermodynamics. For this purpose he imagined a civilization built on a hollow Dyson sphere surrounding a black hole.
- Their civilization is not only literally, but figuratively, built around the black hole, as that is the source of power for them.

Chronology of Black Hole Thermodynamics

- <u>The Black Hole Civilization</u>: A box attached to a spring is filled with thermal radiation and lowered towards the black hole. As it goes down, it winds up the spring. Near the surface of the black hole the box is opened up, so that the radiation falls into the black hole. The lighter box, is now pulled up. The spring is obviously wound up more by the dropping box, than it uses in pulling it up. This generates their energy.



Chronology of Black Hole Thermodynamics

- The second law has often been stated as "There is no such thing as a free lunch". Here, we not only get the "free lunch", of usable energy for unusable energy (like Aladin's old lamps for new), but we also get rid of thermal pollution, making room for more efficient heat engines.
- We not only get the free lunch, we get paid for having it!
- The only way to save the second law is to ascribe an entropy to the black hole. As we feed the black hole our useless thermal energy, say E, we increase its mass by $\Delta M/c^2$. The reduction of entropy of the Universe is by $\Delta S = \Delta M/c^2T$, where T is the temperature of the black hole.
- This must be the increase of entropy of the black hole!

Chronology of Black Hole Thermodynamics

- Here Penrose ran into a problem. What is meant by the temperature of the black hole? He then conjectured that the measure of entropy would be given by the Weyl curvature tensor, which gives the pure gravitational field, without any contamination by matter. The problem is that it is trace-free, so there ca be no scalar constructed from it. Thus one has to go to a quadratic expression in the Weyl tensor.

Chronology of Black Hole Thermodynamics

- People talking of the history now, go into whether Bekenstein or Hawking was first in introducing thermodynamics into black hole Physics, totally ignoring the fact that Penrose had talked of this already in 1968 (at UT Austin, where Bekenstein was working with Wheeler)

— and Bekenstein was in the audience at the time!

Chronology of Black Hole Thermodynamics

- In the early years of this millennium, I was asked by a biographer of Penrose, to give my remarks about him. Apart from the remarks, I pointed out that this work of Penrose had been forgotten. It turned out that Penrose had also forgotten it. After a bit of digging, we found some record of it.
- When I mentioned this to Leonard Susskind, after reading his “Black Hole Wars”, he said it made sense, and explained some things about the history that seemed odd.

Seeing an Object Enter a Black Hole

- According to the standard view of black hole physics, from outside we cannot see an object enter a black hole. This is because the light from it will be infinitely red-shifted and infinitely

time

---

delayed.

Seeing an Object Enter a Black Hole

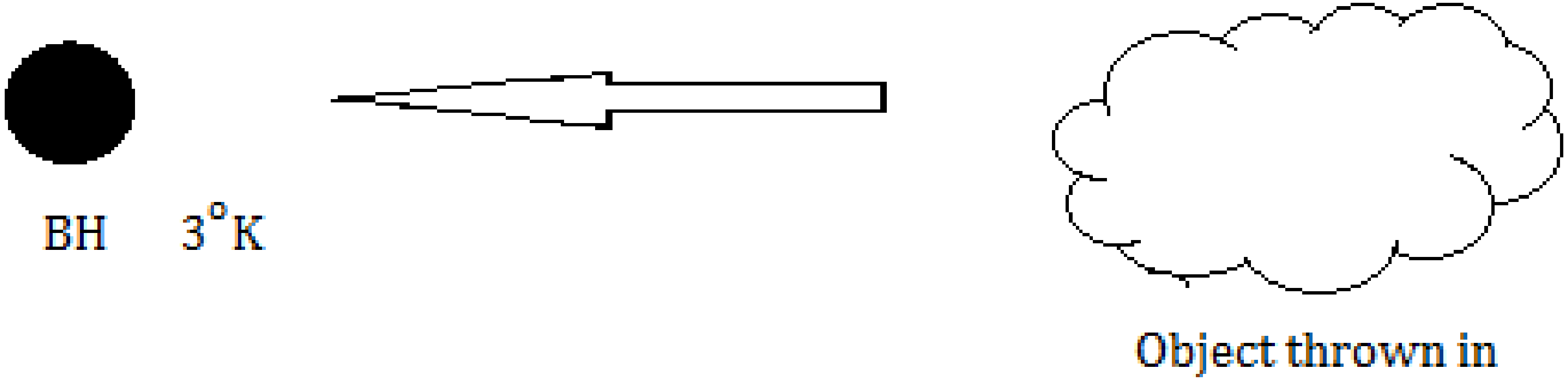
- Now, suppose that there *is* Hawking radiation, and there is a black hole of mass such that it radiates at just above the cosmic microwave background (CMBR) temperature of 2.725 °K, let us say at 3°K.
- The Hawking temperature is

$$T = \frac{hc^3}{16\pi^2 GkM},$$

where M is the black hole mass and k Boltzmann's constant.

Seeing an Object Enter a Black Hole

- Now we throw a stone (that look like a cloud in the diagram) into the black hole. Its mass, *m*, is chosen to be such that the Hawking temperature for the mass (*M* + *m*), is 2.725 ^{o}K.



Seeing an Object Enter a Black Hole

- Before the stone falls into the black hole, the black hole is seen as 0.275 ^oK above the background. Once the stone has fallen into the black hole, the radiation from the hole is effectively "switched off", as it merges into the CMBR and we cease to see the black hole.
- Thus we have seen the stone fall into the black hole, as the shining black hole disappears from view.

Seeing an Object Enter a Black Hole

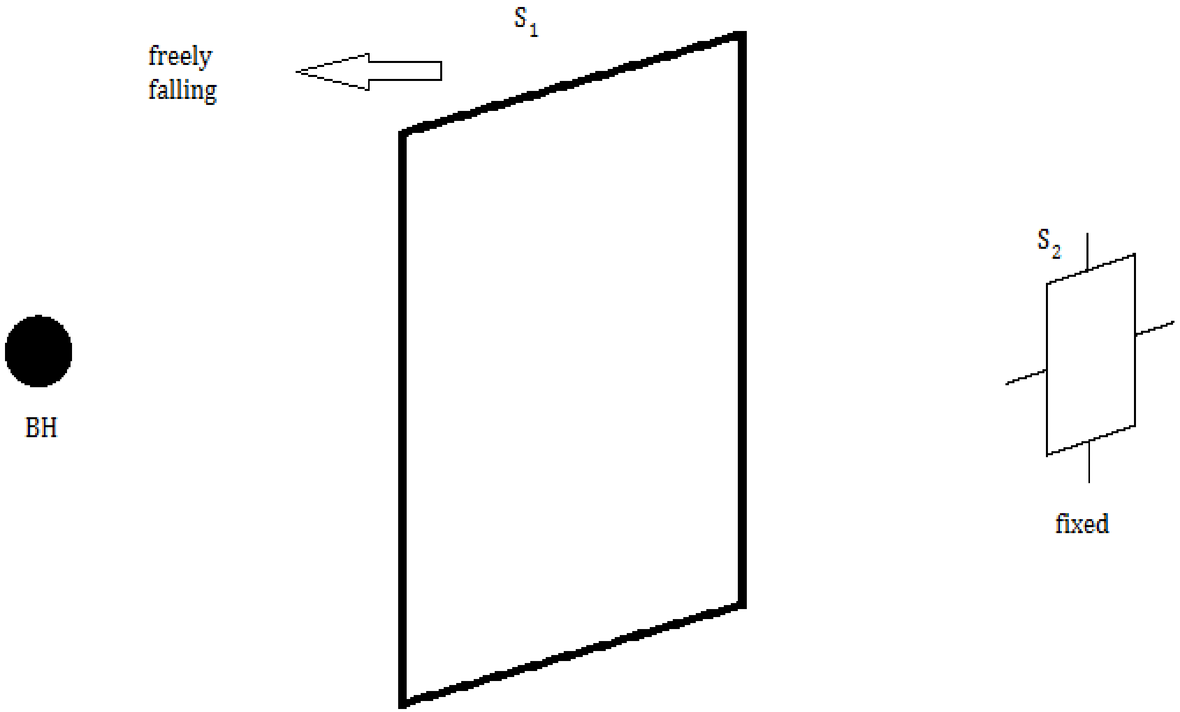
- This brings out the question of when the black hole forms and when the singularity forms. That leads to the question of when the Hawking radiation can start. After all, as the matter falls into the black hole there will be a back reaction, leading to radiation of various frequencies with various time delays. How can we tell when this radiation has stopped and Hawking radiation started?

Detection of radiation in the shadow of a screen

- ✓ An observer falling freely into a black hole will see a flat, or Minkowski, spacetime. As such, the black hole does not seem to exist to that observer, till the time of hitting the singularity. (Of course, tidal forces *would* be felt if the observer is not a point.) As such this observer would not see any Hawking radiation, but a fixed observer would.

Detection of radiation in the shadow of a screen

- ✓ Consider two screens made of photographic emulsion, one of which, S_1, is much larger than the other, S_2, and a black hole, BH, aligned. Take S_1 to be falling freely towards BH, but S_2 to be fixed relative to BH.



Detection of radiation in the shadow of a screen

- ✓ Now the radiation from the black hole is seen by S_1 but not by S_2, despite the fact that S_1 is in the shadow of S_2.
- ✓ How can the radiation seen by S_1 reach it but not pass through S_2?
- ✓ One has to suppose that the radiation does not *travel* from *BH* to S_1, but arrives there *nonlocally*.

Detection of radiation in the shadow of a screen

- ✓If it comes nonlocally, ***WHEN*** does it appear?
- ✓If it is produced by the gravity of the black hole, how valid is the assumption/approximation, that we can ignore quantum gravity and accept the result as correct? Only because it gives meaning to the entropy of black holes, required to save the second law of thermodynamics from the clutches of Penrose's black hole civilization, can it be accepted?

The Carter-Penrose Diagram

- When I went to ICTP in 1975, I was asked to explain "Penrose Diagrams". Now I was writing a paper entitled "Penrose Graphs", a name I had given for the spacetime representation twistor scattering diagrams. I thought that I was being asked about the energy-momentum representation diagrams in MacCallum and Penrose's, *Physics Reports*.
- It turned out that this was a name given to the conformally compactified spacetime diagrams that Penrose used and which had been extensively studied by his student, Martin Walker (with my help). The name had been given by Hawking and Ellis in *The Large Scale Structure of Spacetime*.

The Carter-Penrose Diagram

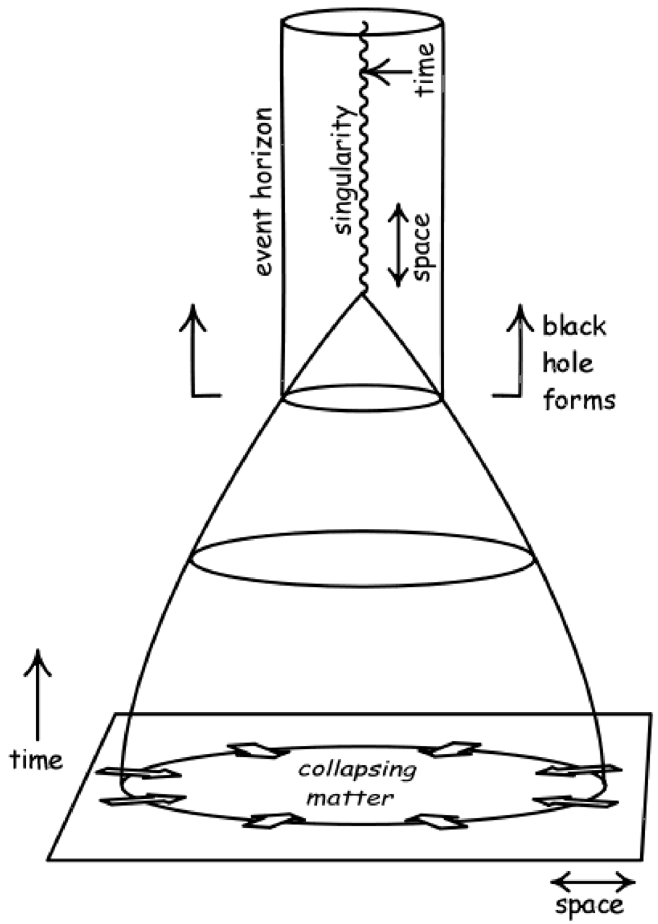
- Penrose had never claimed to be the originator of the diagrams and had been using them. He told Hawking that the diagrams had actually been developed by Brandon Carter. Thereafter, Hawking called them *Carter-Penrose* diagrams.

The Carter-Penrose Diagram

- Since Walker developed the methods for the diagrams further, I would call them Carter-Penrose-Walker diagrams.
- Having worked on these diagrams in helping Walker, once I had seen what was being asked, I had no hesitation in explaining them. I still don't and will go ahead and explain them here.

The Carter-Penrose Diagram

- The usual spacetime diagram for the Schwarzschild black hole is as shown in the adjoining diagram. The vertical axis is time and the horizontal axes, (one dimension suppressed), space. Matter falls into the nascent black hole, till it becomes so dense that even light cannot escape from it. The radius at which this occurs is $2Gm/c^2r$, the Schwarzschild radius, often denoted by r_s.



The Carter-Penrose Diagram

- The problem with this diagram is that the coordinates break down at $r = r_s$. This is seen as the Schwarzschild metric becomes singular:

  $$ds^2 = c^2(1 - r_s/r)dt^2 - (1 - r_s/r)^{-1}dr^2 - r^2(d\theta^2 - \sin^2\theta\, d\varphi^2).$$

  There is also a singularity at $r = 0$.

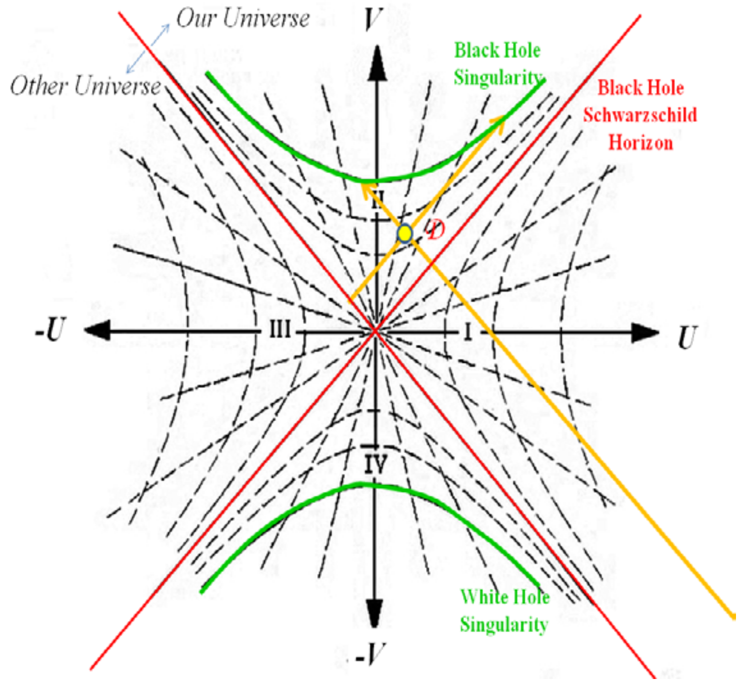
- This singularity is just due to a bad choice of coordinates, like the singularity at $r = 0$ in plane polar coordinates. The solution is to change the coordinates so that this problem does not arise.

The Carter-Penrose Diagram

- The Plasma physicist, Martin Kruskal, developed null coordinates that went through the horizon like a hot knife through butter. Null coordinates are like the usual retarded and advanced times, $u = ct - r$, $v = ct + r$, used for the electromagnetic 4-vector potential.
- For the Schwarzschild metric we define $dr^* = (1 - r_s/r)^{-1} dr$ and use it in place of r. Just this change was introduced *independently* by Eddington and Finkelstein (with a gap of quarter of a century). These still leave the singularity intact.

The Carter-Penrose Diagram

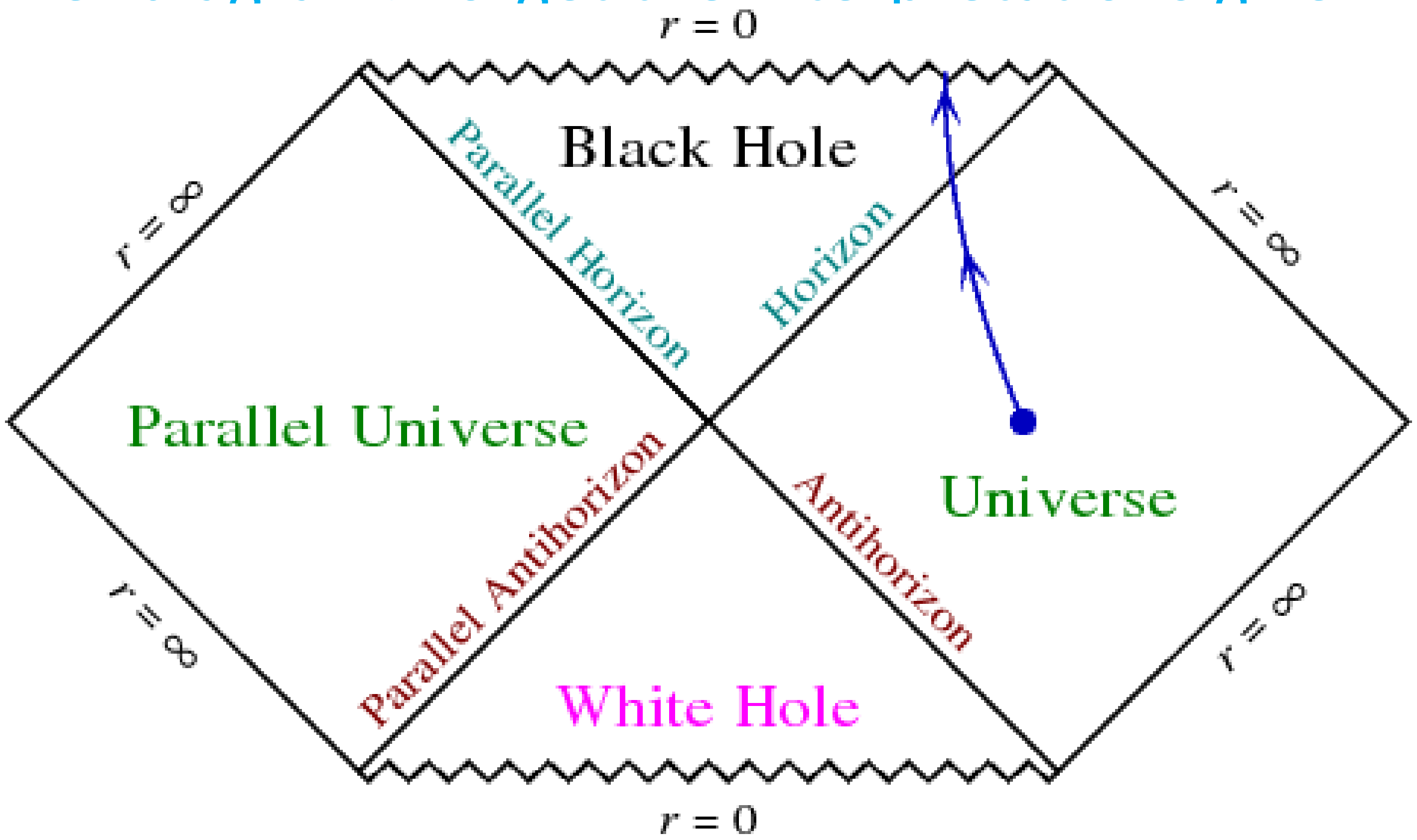
- Kruskal exponentiated and removed the singularity: $U = e^{-u/r_s}$, $V = e^{v/r_s}$, yielding, $ds^2 = (4r_s^3\, e^{-r/r_s}/r)dU\, dV - r^2(d\theta^2 - \sin^2\theta\, d\varphi^2)$, which is nonsingular. This is depicted in the adjoining Kruskal diagram.



- The variables U and V have a doubly infinite range and so the diagram is not easy to see. Carter transformed the variables, so that they had a finite domain. Instead of retaining the null coordinates, we can go back to time and space coordinates. This gives the Carter-Penrose-Walker diagram.

The Carter-Penrose Diagram

- The Carter-Penrose-Walker diagram. Forget the interpretations given in this diagram. The top right corner is *future* timelike infinity, I^+, and the lower right corner *past* timelike infinity, I^-. The extreme right corner is spacelike infinity, I^0. The lines joining I^- to I^0 and I^0 to I^+ are past and future null infinity, $\mathcal{J}^-$ and $\mathcal{J}^+$.

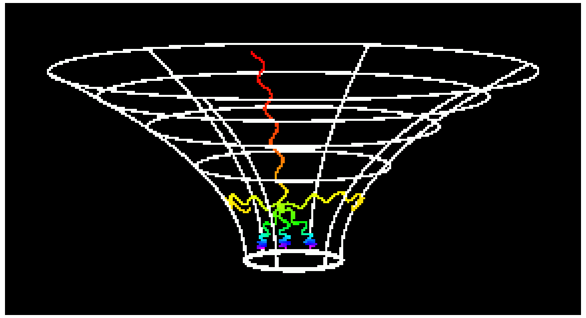


The Carter-Penrose Diagram

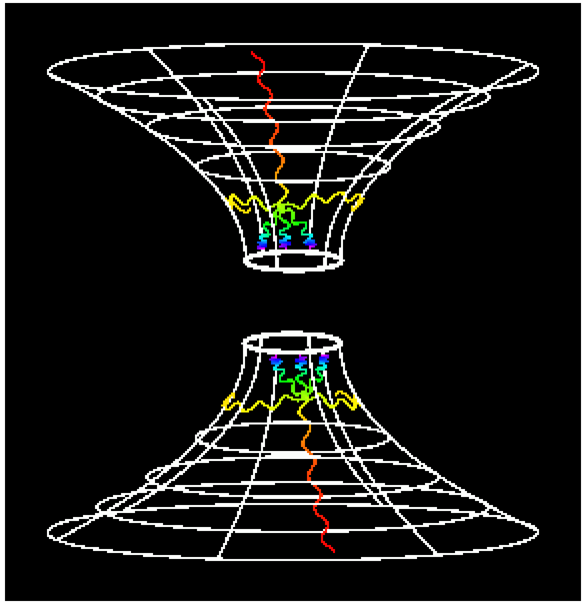
- The Carter-Penrose-Walker diagram:
- I^- is at $t = -\infty$, $r = 0$; I^+ is at $t = +\infty$, $r = 0$; I^0 is at $t = 0$, $r = +\infty$; $\mathscr{I}^-$ is at $r = -ct = \infty$ and $\mathscr{I}^+$ is at $r = ct = \infty$.
- The extreme right rhombus is the usual Universe outside the black hole; the upper triangle is the black hole as seen by people falling in in the future; and the lower triangle the black hole as it was in the past. The million dollar question is “what is the rhombus on the left”? This region had been required by Einstein and Rosen noting that all geodesics (shortest paths) must either go on to infinity or end at a singularity. This was not satisfied without this region.

The Carter-Penrose Diagram

- In the diagram below the spacelike geodesics end up nowhere.

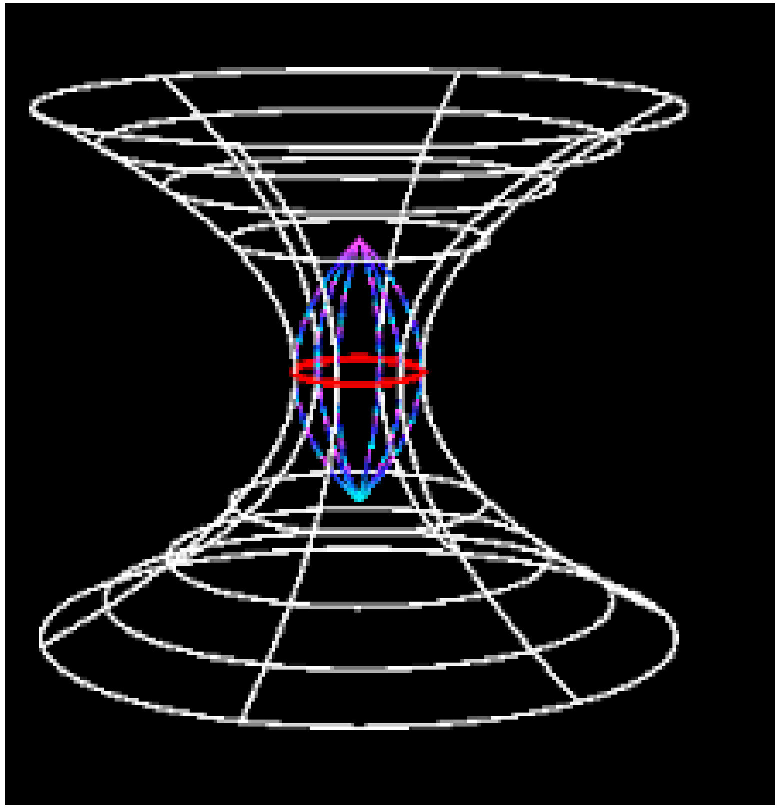


- To complete it we need to put two of them together upside down.



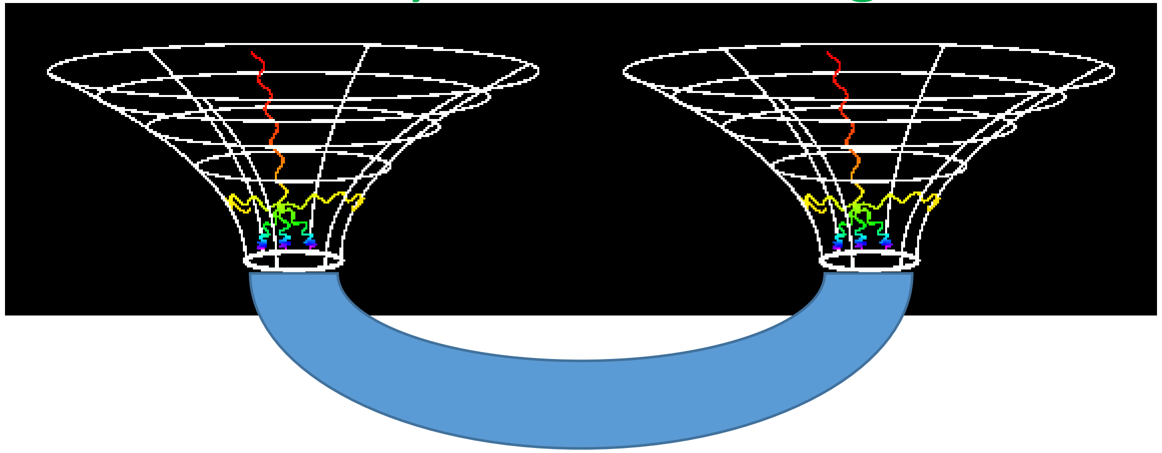
The Carter-Penrose Diagram

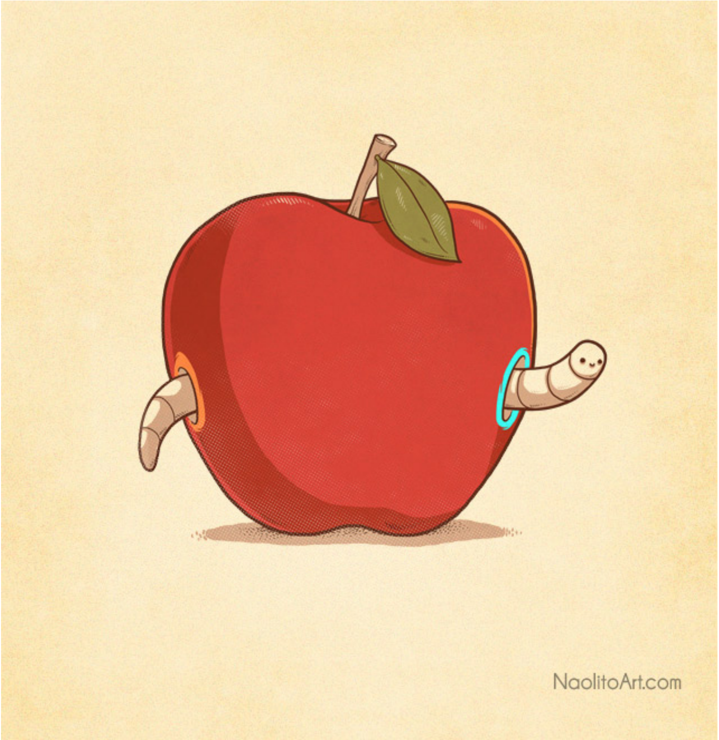
- The joined diagram then becomes as seen here. The joining part is called the Einstein-Rosen bridge. The upper part corresponds to the spatial section of the usual Universe, and the lower part is called the maximal extension, and is the fourth region in the diagram.



The Carter-Penrose Diagram

❖If instead we put them together as below and join the two ends at the bottom by a tunnel, we get a *wormhole*.





❖Remember that since Newton's story of how he arrived at the theory of gravity was being hit on the head by an apple, and *not* by following up on Hook's suggestion, gravity is associated with apples.

Ambiguity in Labeling CPW Diagram

- Let us go back to the CPW Diagram and my claim about an ambiguity in labeling it.
- If the question of the significance of the fourth region was a $ 1,000,000 question, the $ 64,000,000 question is what do the right vertices represent?

Ambiguity in Labeling CPW Diagram

- When I was looking into this in 1988, it seemed to me that there was no unique way to assign the right labels to the left side. It depended on how I viewed the spacetime. I tried looking up the relativist's bible, *Gravitation* by Misner, Thorne and Wheeler, but they left it unlabeled. I went to the other relativist's book by Hawking and Ellis, but *they* left it out. I tried writing to Wheeler and got no response. I tried writing to Roger Penrose and got no response.

Ambiguity in Labeling CPW Diagram

- There are two ways of looking at a spacetime: one is from the point of view of the observer, which is a local way; and the other is from the point of view of the spacetime as a whole, which is a global way. In both cases we have the whole spacetime by the end of it, so there is no way, a priori, to decide which to use.

Ambiguity in Labeling CPW Diagram

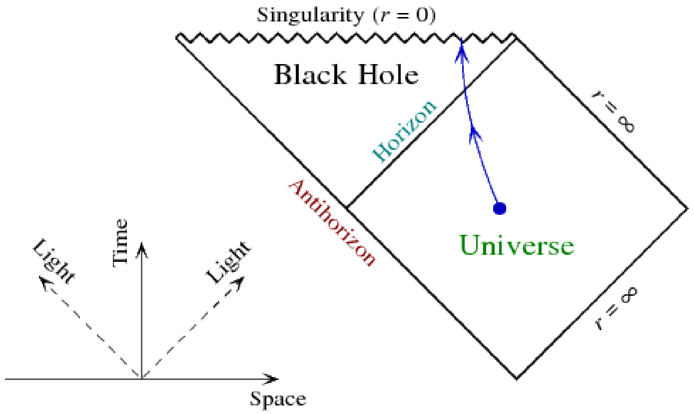
- For the former, we need to define our preferred class of observers and then take the collection of all of them to give the spacetime. What I considered was physically, the class of observers falling freely from infinity, starting at rest. We have showed that this corresponds to breaking the spacetime into a sequence of spacelike hypersurfaces of zero intrinsic curvature, i.e. with the 3-d Riemann curvature zero.

Ambiguity in Labeling CPW Diagram

- For the latter I took the breakup by hypersurfaces of constant mean extrinsic curvature. (The extrinsic curvature tensor is $K^i{}_j = -n^i{}_{;j}$, where **n** is the unit normal vector to the hypersurface, and the mean extrinsic curvature is the trace of this tensor, $K = -n^i{}_{;i}$.)

Ambiguity in Labeling CPW Diagram

- For the former we get the picture built up from the hypersurfaces as on the left. The left side of the diagram is missing. Now rotate it and you get the left side. Adjoin and you get the whole diagram. Here the arrow of time is upside down on the left of the CPW diagram. Neither side talks to the other. Obviously, the upper left corner is I^+ and the lower left corner is I^-, with the null lines reversed as well.



$\mathcal{J}^+$

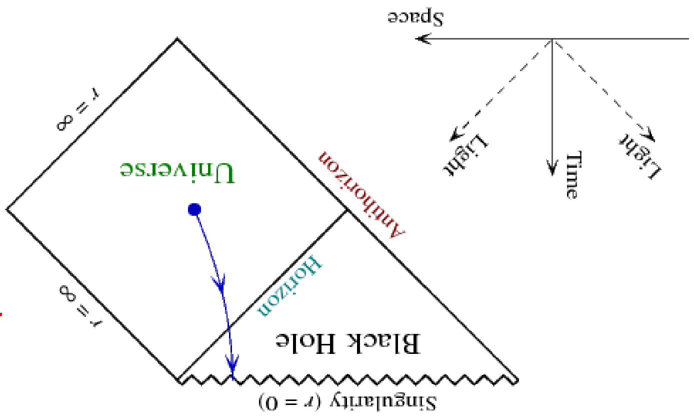
I^0

$\mathcal{J}^-$

$\mathcal{J}^-$

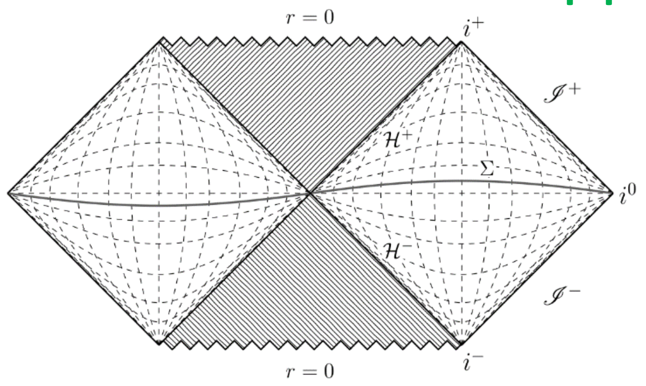
I^0

$\mathcal{J}^+$



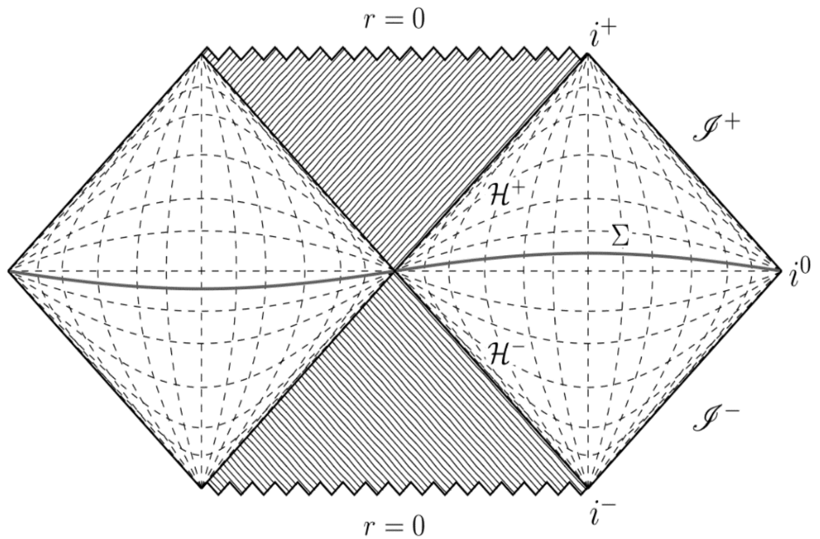
Ambiguity in Labeling CPW Diagram

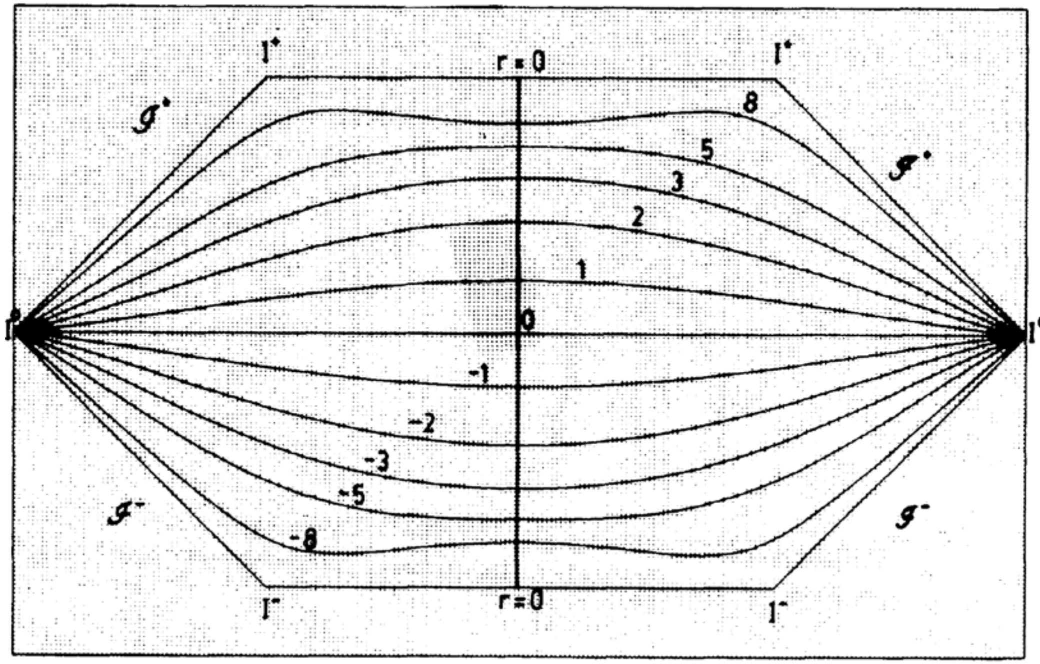
- Now consider the global way of defining our spacetime breakup into space and time. This time was defined by Jim York and thus it is called "York time". A generic hypersurface is shown in the diagram below on the left. The breakup obtained by my students A. Pervez, A.A. Siddiqui and I is given on the right. Clearly, here the lower left corner is I^- and the upper I^+. The arrow of time is up both sides. Here, $\mathscr{J}^-$ is the lower one and $\mathscr{J}^+$ is the upper one.



Ambiguity in Labeling CPW Diagram

- The Diagrams: The left is a generic spacelike hypersurface and the right is the *K*-slicing (slices of constant *K*).





Ambiguity in Labeling CPW Diagram

- The choice is significant in attempts to do Quantum Field Theory in the chosen background. When we take equal time commutation relations, the choice of what the equal time taken is, is obviously of vital importance. If we go with the local definition made global, as we had done, half the diagram is missing and this will create spurious fields. That was seen by Fulling using the Rindler wedge. His paper was titled "Nonuniqueness of Canonical Field Quantization in Riemannian Space-Time". If, instead we take the global viewpoint, the full maximally extended spacetime is taken. There is no missing part leading to spurious additional fields.
- Hawking took the former view.

Ambiguity in Labeling CPW Diagram

- If, as I am implying, Hawking's result is spurious, how does it give the entropy? While Penrose never gave the black hole entropy in terms of the geometrical quantities found by Bekenstein and Hawking, even his result is consistent with that of the other two, that the entropy is proportional to the black hole area.

Ambiguity in Labeling CPW Diagram

- I am suggesting that the total result may be that the contributions of all other fields and quantum gravity *exactly* cancel out. In that case, essentially, performing half the calculation may provide us with the entropy in a manner similar to that of getting the string tension using D'Alembert's principle.

Ambiguity in Labeling CPW Diagram

- *But* ***why*** should it cancel instead of adding? Well, this goes back to the observation by Maxwell, that electromagnetic force, and hence energy, goes in the opposite way to gravity, as like electromagnetic charges repel but gravitational "charges" attract.

Ambiguity in Labeling CPW Diagram

- When I read Susskind's fascinating book *The Black Hole War: My Battle with Stephen Hawking to Make the World Safe for Quantum Mechanics*, I was thrilled with his exposition of the problem and his proposed resolution.

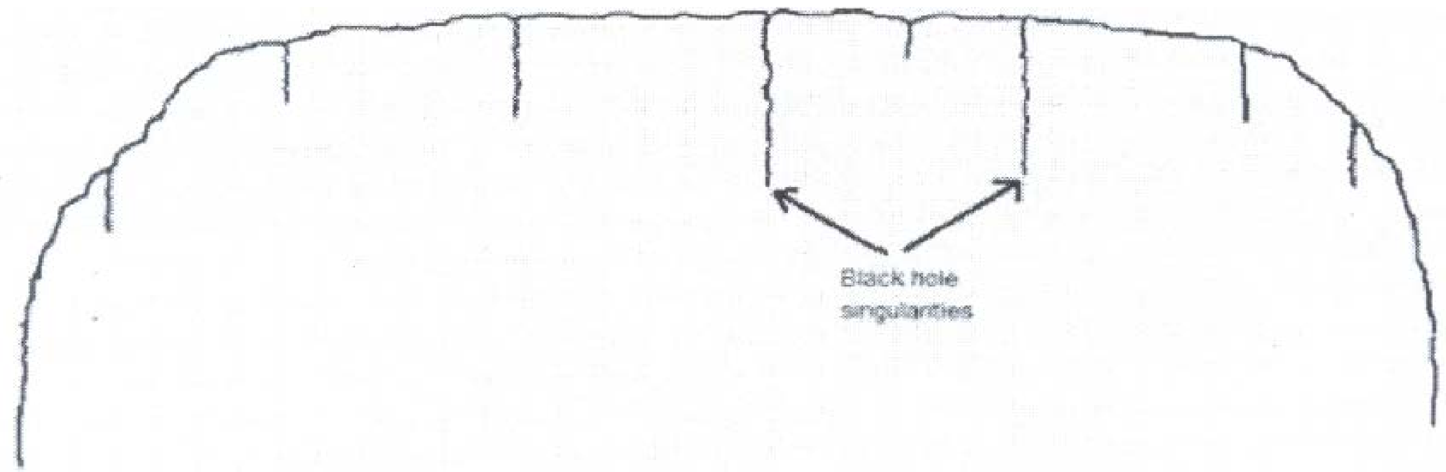
Ambiguity in Labeling CPW Diagram

- I wrote to him, congratulating him on his superb book and pointing out the error in the history I mentioned earlier. At the end I remarked about my work with Wheeler and how I thought I had an alternate, nearly classical, resolution to the problem of information loss and how Wheeler was not ready to look at it because of his "religious belief" in a closed Universe. He said I should revive the work — so here it is.

The Qadir-Wheeler Suture Model

- ✓Roger Penrose had conjectured that the black hole and final singularity would be simultaneous, and if the Universe is flat or open, then the conformally compactified version of the Universe would have a simultaneous end with the black hole. He presented this via a picture of a cave with stalactites.

Big Crunch

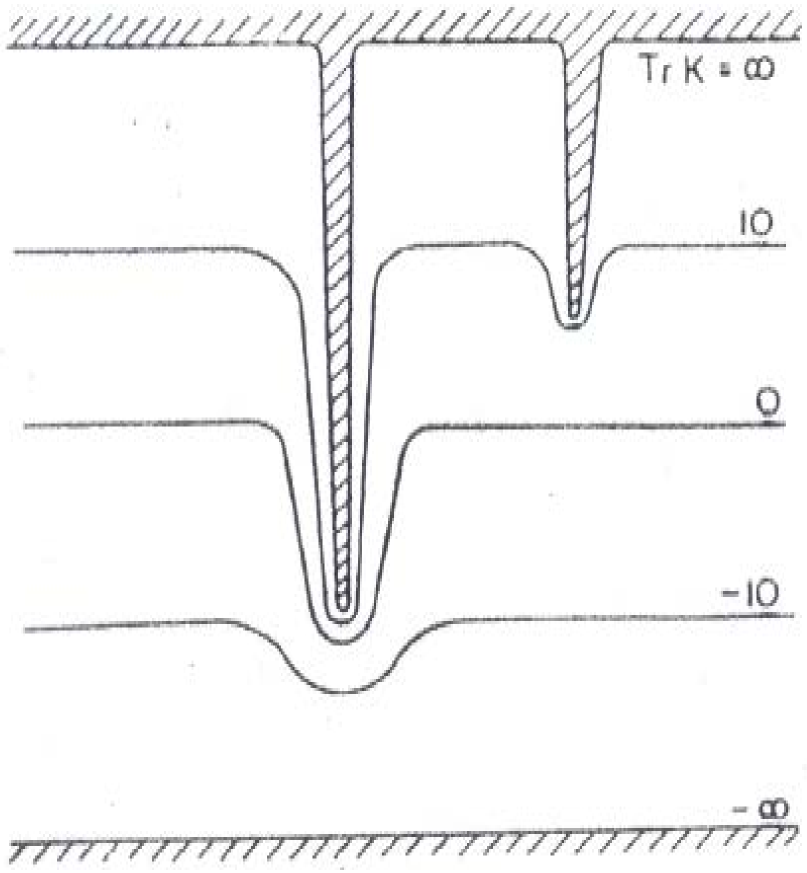


The Qadir-Wheeler Suture Model

- ✓When one says “simultaneous with”, one means that both lie on the same spacelike hypersurface. Thus one is claiming that a sequence of spacelike hypersurfaces with the limit as the big crunch will have the black hole singularity in the same limit. This is clearly a global view and requires the constant mean extrinsic curvature hypersurfaces.

The Qadir-Wheeler Suture Model

- ✓As Wheeler liked to put it, the sequence would fit on to the singularity like a glove with the black hole singularities taking the role of the fingers and the glove fitting on to the fingers.



The Qadir-Wheeler Suture Model

- ✓We tried a Schwarzschild lattice universe and proved the result, but in that case the singularity never formed, so the *point* was not proved. We tried putting a dust shell collapsing, starting at the phase of maximum expansion and proved the result, but the shell or cloud stuck out from the Universe at the Big Bang. Again we had proved the result but not the point. It was the old Goldilocks problem, the first was too “soft” and the second too “hard”.
- ✓So then we went to the Baby Bear’s chair and porridge and bed.

The Qadir-Wheeler Suture Model

- ✓ That was our suture model, which I now explain. We took two closed Friedmann model universes of different densities at the phase of maximum expansion. As such, they would have different life-spans, the denser one evolving faster and the rarer one slower. We cut out one part of the denser and another piece of the rarer, so that the masses are equal, and required that they fit together perfectly in the limit as we approach the big bang. Now, as the universe parts evolve (at different rates) a gap opens up, which is given the Schwarzschild geometry.

The Qadir-Wheeler Suture Model

✓The metric for the closed Friedmann model is

$$ds^2 = c^2dt^2 - a^2(t)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)], \qquad (*)$$

which can be written as

$$ds^2 = c^2a^2(\eta)\,[d\eta^2 - d\chi^2 - \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)], \qquad (\#)$$

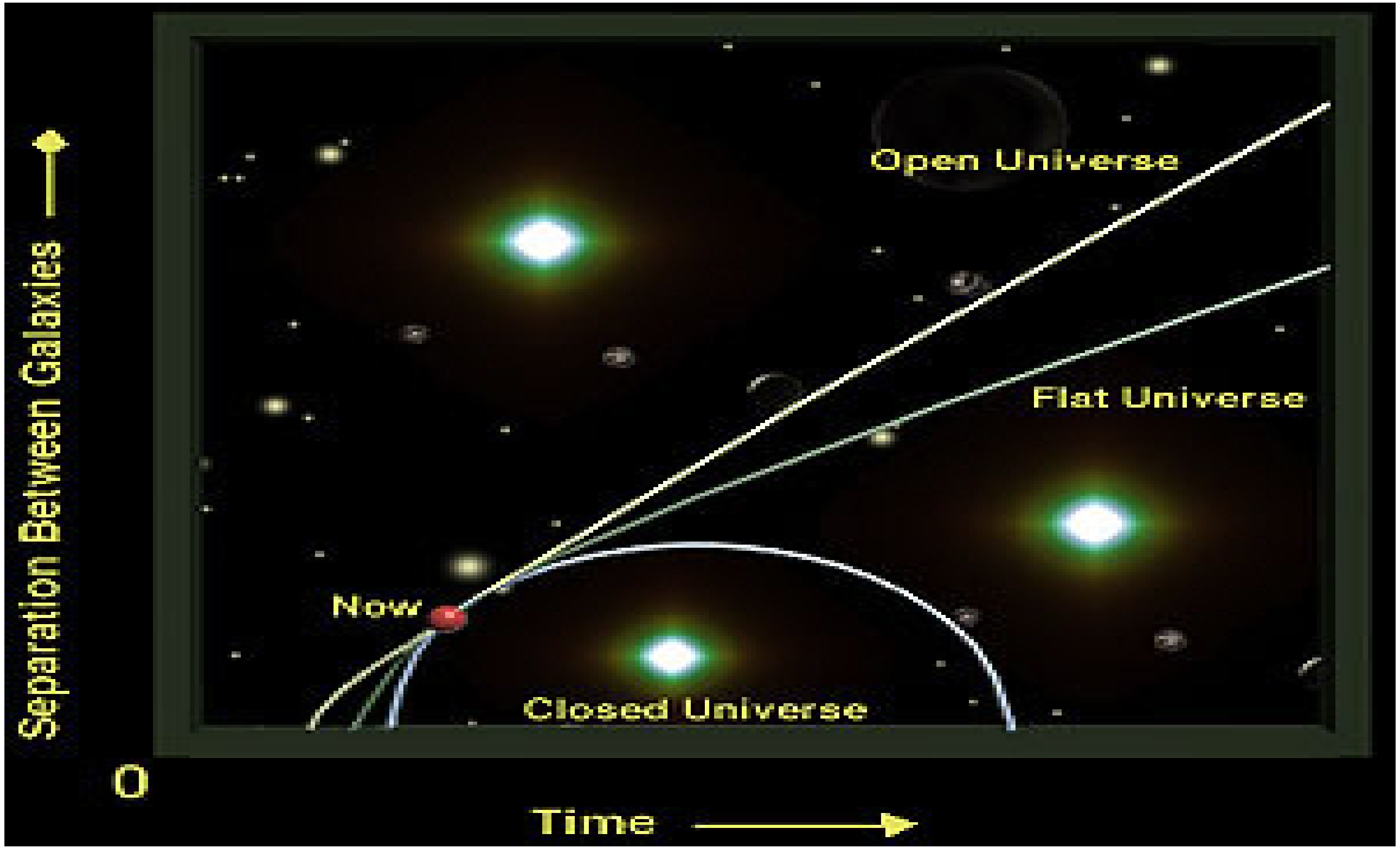
where

$$a(t) = (a_o/2)(1 - \cos\eta)\ ;\ t = (a_o/2)(\eta - \sin\eta)\ .$$

Thus $0 \leq \eta \leq 2\pi$ and so $0 \leq a \leq a_o$ and $0 \leq t \leq a_o\pi$. At $t = a_o\pi/2$, $a = a_o$ and at $t = a_o\pi$, $a = 0$.

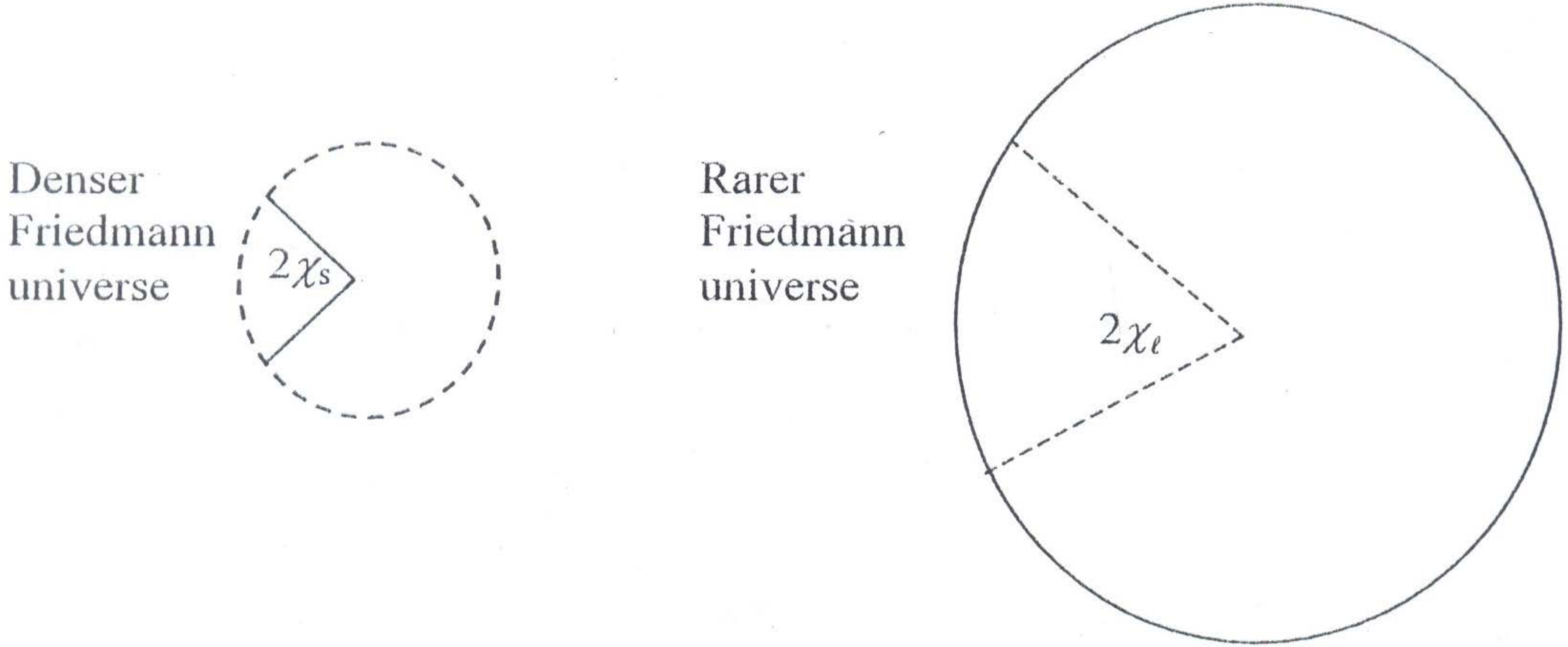
The Qadir-Wheeler Suture Model

- ✓This figure misleads. The three models start at the same Big Bang, but it is good enough otherwise.



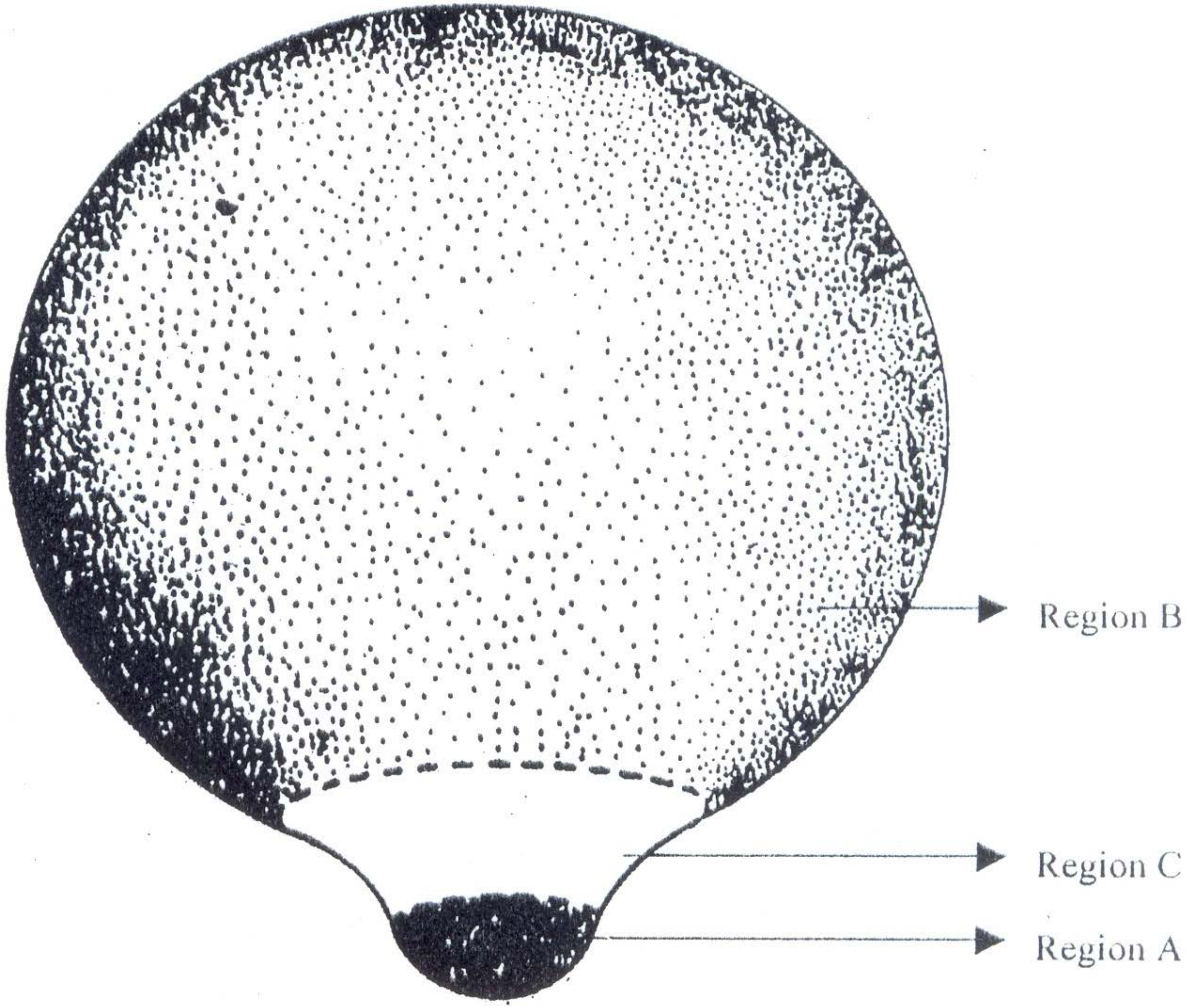
The Qadir-Wheeler Suture Model

✓The Schwarzschild region is the suture joining the two parts together.



The Qadir-Wheeler Suture Model

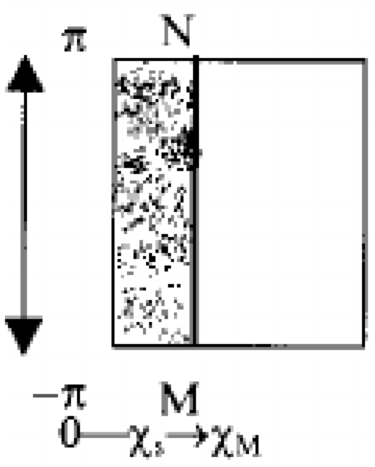
- ✓The resulting model is as shown in the diagram: Region A is the denser closed Friedman section; Region B, the rarer; and Region C the Schwarzschild suture.
- ✓We can now follow its evolution from beginning to end.

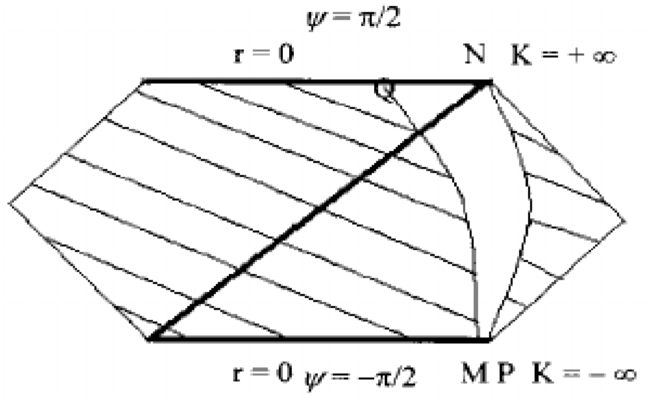


The Qadir-Wheeler Suture Model

✓The corresponding CPW diagram is given below, in three parts.

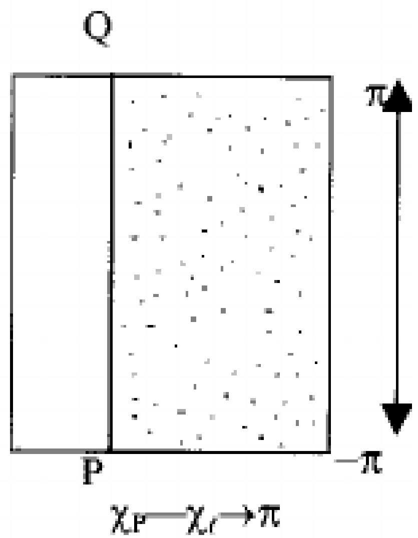
Region A





Region C

Region B



The Qadir-Wheeler Suture Model

- ✓Dealing with a cut-and-paste model needs special care. Not only the coordinates, but the geometry is different in each part. Further, the usual procedure in General Relativity, of using analytic transformations of the coordinates, to go from one domain to another, is not available here. We have to use invariant geometric measures at, and of, the boundary. Thus we need to measure the circumference of the boundary on either side of the join of Regions A &B and of Regions B & C. We need to state precisely what our measure of mass is, and require that the mass, so defined, of Region A is the same as the mass cut out from the universe model to get Region C.

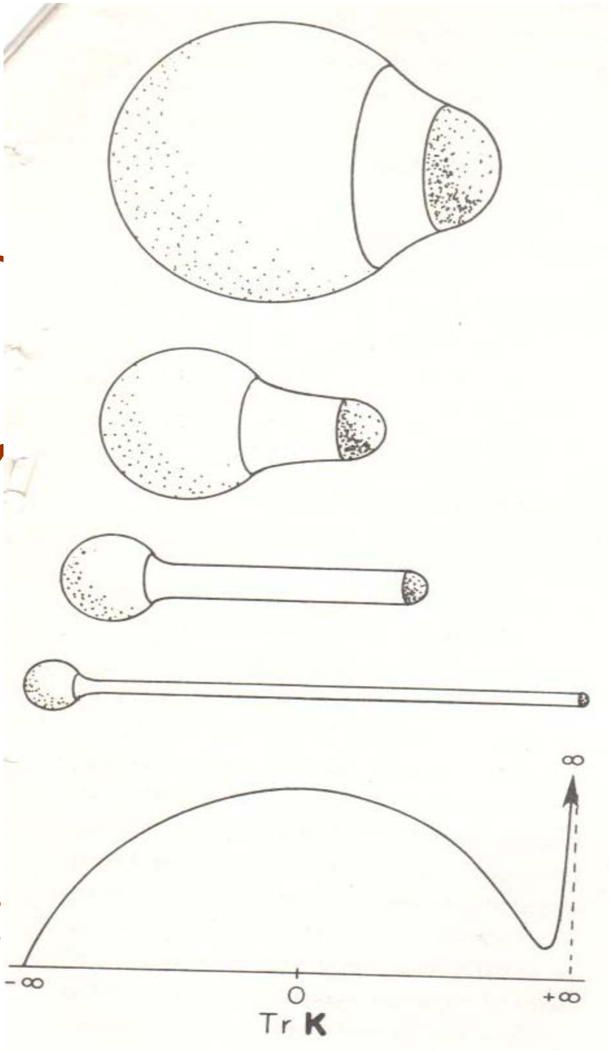
The Qadir-Wheeler Suture Model

- ✓The connection from the inner Friedmann, with variables (η, χ), to the Schwarzschild, with variables (t, r), is given by $R_k = a_k sin\chi_k$, where the subscript $_k$ is for "smaller", if it takes the value $_s$, or "larger", if it takes the value $_l$ and R_k is the r value at the boundary and χ_k, the hyperspherical angle at the boundary. We then require that the circumference of the boundary be the same at all times, but has to be described in terms of the two different coordinates. This yields the junction conditions at both the boundaries to be

$$R_k = (a_{ko}/2) \sin \chi_k (1 - \cos \eta_k).$$

The Qadir-Wheeler Suture Model

- ✓The evolution of the hypersurfaces for how the Universe "size" behaves with *K* is shown in the adjoining diagram. The top one is at the phase of maximum expansion. As the inner universe contracts the outer will first expand, but by the time that the black hole has formed, the outer one has started its collapse. As time goes on, both parts more or less disappear and only the Schwarzschild region contributes. The proper distance between poles increases till the phase of maximum expansion of the outer region, contracts and then, later, goes shooting off to infinity.



The Qadir-Wheeler Suture Model

- ✓Though people should know this, the popular presentation of black holes as collapsing to a point at the centre misleads them. It is well to bear in mind what Wheeler said: "$r = 0$ is not *where* the black hole collapses, it is *when* it collapses". It is the end of *time* and not *space*. Spatially, the collapse is onto a line.

The Qadir-Wheeler Suture Model

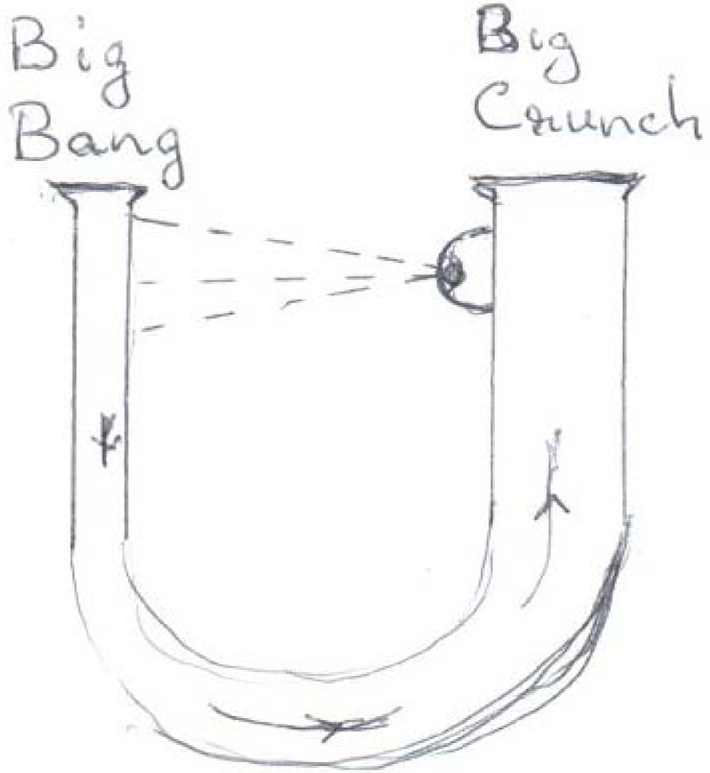
- ✓The model is not limited to a single black hole but can be used for a number of them of varying sizes. Regardless of whether the model is closed, flat or open (*k=+1, 0, -1*), the end of the Universe is unchanged — it is a mess of very thin corridors connected at nodes. And the lengths of the corridors go shooting off to infinity.
- ✓The Universe is *not* on a knife-edge (of *k =0*) in any case!

The Nearly Classical Resolution

- So what, if anything, does all this have to do with information loss? Recall that Penrose had not limited his conjecture to a closed model but included open and flat models. I thought of what might happen for one of those models and took my idea to Wheeler. But Wheeler believed that the Universe *had to be* closed, because only so could intelligences in the Universe create it by observing its beginning. (This is tied up with his ideas of observer participation for Quantum Theory.)

The Nearly Classical Resolution

- The basis of Wheeler's philosophy. The "U" represents the Universe, and time goes along the U from left to right. By the end of the Universe, the intelligences of the time will have observed its beginning, and by observer participation, thus *created* the Universe.



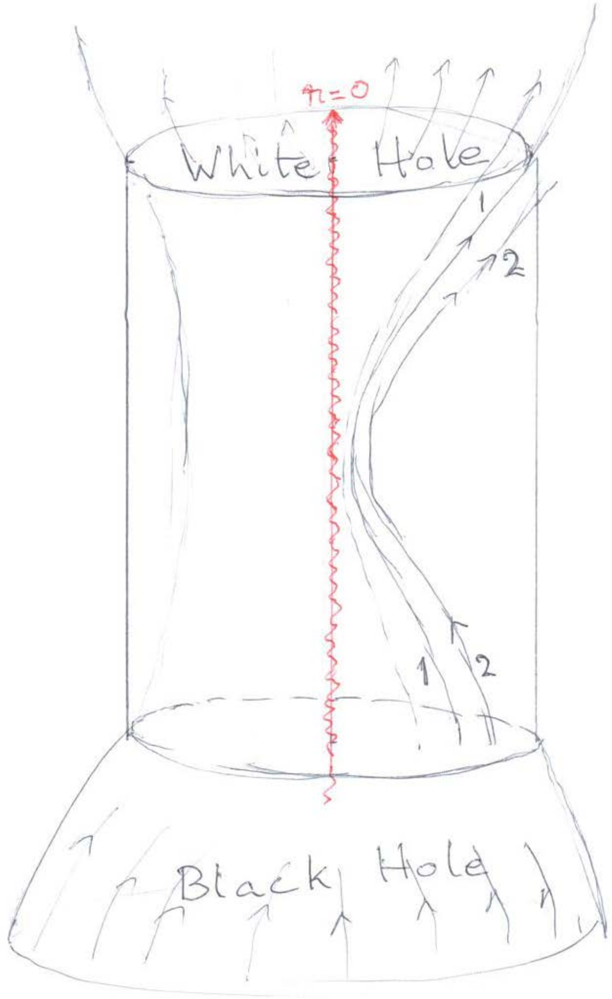
- Discouraged, I left it alone. However, encouraged by Susskind, here is what I had come up with.

The Nearly Classical Resolution

- If the end of the black hole is simultaneous with the end of the Universe and the Universe never ends, *neither will the black hole,* i.e. the black hole singularity never forms!
- Since General Relativity is time symmetric, given enough time, the incoming matter must come out, so that the erstwhile black hole has become a white hole. The matter that went in, will come out, but *in reverse order*. To see why, think of the world lines of the matter traced out time symmetrically — and "the first shall be last".

The Nearly Classical Resolution

- A spacetime picture of the black hole collapse converting to a white hole by time reversal invariance symmetry. As Wheeler had a religious belief in a closed Universe, he was not ready to even think of what would happen to a flat, or open Universe.



The Nearly Classical Resolution

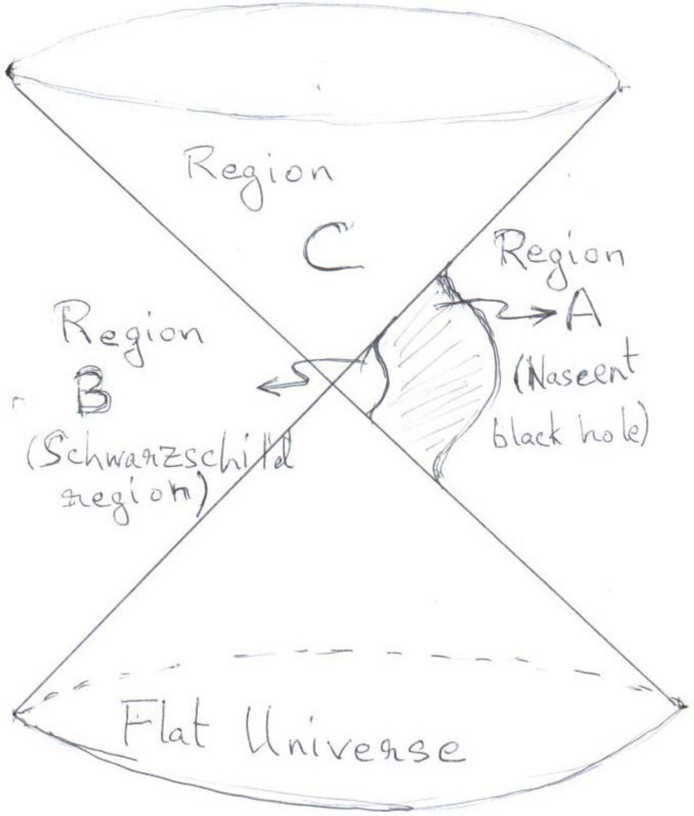
- The weak link here is “given enough time”. How much time? Must we wait for an infinite time? If not, how does it decide to just turn over? To put it another way, what makes it bounce? There must be some mechanism for it.
- Here, without knowing it, In 1978 I used Bekenstein’s argument. There will be a minimum size, i.e. volume, that Heisenberg’s uncertainty principle will allow for any given mass black hole. This is more or less what he used when he got the entropy of the black hole.

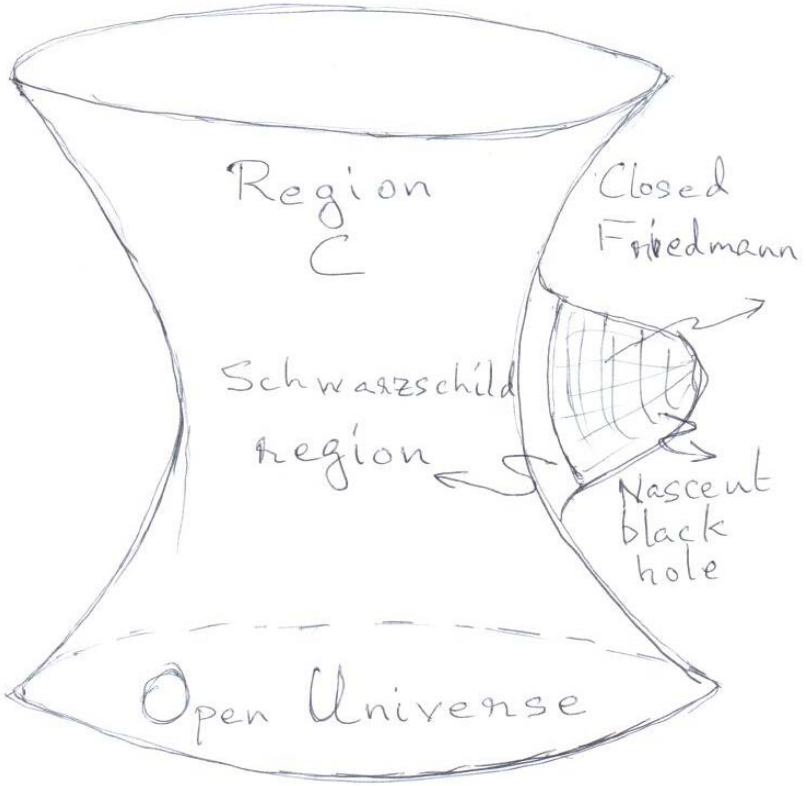
The Nearly Classical Resolution

- To proceed with this work for an open or flat Universe, it is necessary to construct the open and flat models. It is not, a priori, obvious that this can be done. After all, the black hole collapses to the singularity in a finite proper time, while the outside Universe never does so. The problem could lie with the "religious belief" that a one dimensional line singularity is physically attainable. Mathematically, we can try to simply follow Penrose's suggestion of using a transformation to bring "infinity" to a finite place. The other problem could still arise, of being able to manage the junction conditions adequately.

The Nearly Classical Resolution

- The diagrams for the flat and open suture models are given below:





The Nearly Classical Resolution

- The metric for the flat and open Friedmann Universe models change the form of a and the dependence on χ in (*) and of t and a on η in (#). For the flat case we get $a \sim t^{2/3}$ or η^2, and χ instead of $\sin \chi$. Thus, $0 \leq \chi \leq \infty$ for this case. For the open we have

  $a(t) = (a_o/2)(\cosh \eta - 1)$; $t = (a_o/2)(\sin \eta - \eta)$ and $0 \leq \chi \leq \infty$.

- To avoid the problem of the infinite domain of χ and η, we define new variables $\chi' = \tan^{-1}\chi$ and $\eta' = \tan^{-1}\eta$, with finite domains, $-\pi/2$ to $\pi/2$. We now get junction conditions as before, but with the appropriate change of variables and functions.

The Nearly Classical Resolution

- Aneela Naheed and I got the junction condition for the open Universe as

  $R_l = (a_{ko}/2)\, sinh(tan\, \chi_l')[cosh(tan\, \eta_l')-1].$

  Similarly, one gets the flat junction condition

  $R_l = (a_{ko}/2)\, tan^2\chi_l'\, tan^2\eta_l'.$

- What remains is to proceed with the foliation for both these models and then put in the Bekenstein condition for the minimum “thickness” of the collapsing spheres, after reaching which the model would “bounce” as explained before.

Problem with 2-d Black Holes

- While doing the computing for the foliation, I had found that as one approached the singularity two complications arose: greater sensitivity leading to numerical instabilities; and more and more steps of computation. I first thought that the attempt to deal with the former led to the latter problem.

Problem with 2-d Black Holes

- I later realized that there was more going on than merely the numerics. The proper length along the hypersurface of the Schwarzschild part was increasing as one approached the singularity. While Wheeler was pleased to see this feature of our model, he wanted to see it proved analytically.

Problem with 2-d Black Holes

- This was done much later. Atiya-tul Hussain and I showed analytically that the proper length of the corridor, $\Delta s \sim K^{1/3}$, as it opens. Then Azad Siddiqui and I did the calculation for the pure Schwarzschild black hole and showed that $\Delta s \sim K^{1/3} \ln K$. Why the difference? The extreme corners are cut out for the suture model but incorporate the logarithmic factor for the Schwarzschild.

Problem with 2-d Black Holes

- How about the volume? As the circumference *shrinks* with the York time by a factor proportional to K, the volume, $V \sim K^{-1}$. Thus we do, indeed, get a collapse with time for the suture model, despite the proper length diverging.

Problem with 2-d Black Holes

- For the Schwarzschild geometry, $V \sim K^{-1} \ln K$. The extra factor comes because as we approach the line singularity the proper distance between the part cut off by the suture model and the corner of the Schwarzschild singularity diverges logarithmically. The mild increase in the proper volume due to the logarithm is totally swamped by the sharp decrease due to the reduction of the "circumference", the S^2 part.

Problem with 2-d Black Holes

- So far no problem has appeared. However, Azad and I were not satisfied with leaving well enough alone and proceeded to consider any (n) dimensional spacetimes. In that case as is easily seen $V \sim K^{1-2n/3} \ln K$, since the “radial” parameter of the black hole, $r \sim K^{-1}$ and the hyper-volume of a hyper-sphere in ($n+1$) dimensions $\sim r^n$. Now putting in the Δs we get this result.

Problem with 2-d Black Holes

- For $n = 3$, we get the previous result, for higher n it goes more sharply to zero, for $n = 2$, we get V going somewhat slowly to zero,

  but for a 2-d spacetime, $n = 1$, $V \sim K^{1/3} \ln K$, and the volume *diverges*!

- There is no collapse but an explosion!

Problem with 2-d Black Holes

- For the Schwarzschild geometry, $V \sim K^{-1} ln\ K$. The extra factor comes because as we approach the line singularity the proper distance between the part cut off by the suture model and the corner of the Schwarzschild singularity diverges logarithmically. The mild increase in the proper volume due to the logarithm is totally swamped by the sharp decrease due to the reduction of the "circumference", the S^2 part.

Problem with 2-d Black Holes

- So far no problem has appeared. However, Azad and I were not satisfied with leaving well enough alone and proceeded to consider any (n) dimensional spacetimes. In that case as is easily seen $V \sim K^{1-2n/3} ln\, K$, since the "radial" parameter of the black hole, $r \sim K^{-1}$ and the hyper-volume of a hyper-sphere in $(n+1)$ dimensions $\sim r^n$. Now putting in the Δs we get this result.

Problem with 2-d Black Holes

- For $n = 3$, we get the previous result, for higher n it goes more sharply to zero, for $n = 2$, we get V going somewhat slowly to zero,

  but for a 2-d spacetime, $n = 1$, $V \sim K^{1/3} \ln K$, and the volume *diverges*!

- There is no collapse but an explosion!

Conclusion

✓ By Penrose's argument, we *need* that black holes have entropy. Bekenstein provided a formula for the black hole entropy, up to a constant. Hawking provided a temperature corresponding to the entropy and giving the constant. However, I pointed out some doubts about Hawking radiation neglecting quantum gravity.

Conclusion

- ✓It is claimed that wrapping a 2-d black hole in a super-membrane also gives the entropy. However, we have seen that 2-d black holes are not black holes at all — instead of collapsing, they explode.
- ✓So, if we are left without the branes to get the entropy, and we do not have a Hawking temperature to give meaning to the entropy, can we talk of the entropy and provide a formula for it?

Conclusion

- ✓If we have no dynamics, can we talk of string tension? Yes, we can — using D' Alembert's principle. Similarly, it could be that the quantum gravity effect would exactly balance the effect of quantizing other fields in the curved background.
- ✓But if we have no Hawking radiation, would it be impossible for anything to come out of the black hole and the black hole to disappear? Using the suture model for open and flat Friedmann models, the black hole would become a white hole.