

List of Topics for Linear Algebra:

1. Linear equations and matrices, reduction to row echelon form.
2. Vector spaces:
 1. Vector spaces, subspaces, quotient spaces.
 2. Linearly independent sets.
 3. Linear transformations,
 4. kernel and image,
 5. projections (idempotent linear operators),
 6. the set of linear transformations between two vector spaces forms a vector space.
 7. Bases and dimension for finite dimensional vector spaces.
3. Matrices and linear transformations between finite dimensional vector spaces:
 1. The matrix of a linear transformation with respect to a choice of bases,
 2. similarity of matrices and change of basis for linear transformations.
 3. The inverse of a matrix,
 4. the determinant of a square matrix,
 5. the characteristic polynomial,
 6. the minimal polynomial,
 7. eigenvectors,
 8. eigenvalues.
 9. Diagonalizability,
 10. Cayley-Hamilton theorem.
 11. Rank + nullity = dimension of domain.
4. Finite dimensional inner product spaces:
 1. The standard positive definite inner product on real n -space,
 2. length and angle,
 3. Gram-Schmidt orthogonalization.

Basic Analysis

1. Metric Spaces
 1. Convergence of sequences in metric spaces
 2. Cauchy sequences
 3. completeness
 4. contraction principle
2. Topological spaces
 1. continuous maps
 2. Hausdorff spaces

3. compactness
 4. connectedness
3. The real numbers
 1. The real numbers as a complete ordered field
 2. closed bounded subsets are compact
 3. intermediate value theorem
 4. maxima and minima for continuous functions on a compact set
4. Differentiation
 1. Differentiation of a function in one real variable
 2. Mean Value Theorem
 3. L'Hopital's Rule
 4. Taylor's Theorem with error estimates
5. Riemann integration of functions in one real variable
 1. Definition
 2. Riemann integrable functions
 3. integration and anti-differentiation
6. Sequences and series of functions
 1. power series and interval of convergence
 2. uniform convergence of sequences of functions
 3. uniform convergence and integration
7. Differential Calculus for functions from n -space to reals and reals to n -space
 1. Parametrized curves
 2. tangent vectors
 3. velocity
 4. acceleration
 5. partial derivatives
 6. directional derivatives
 7. the gradient
 8. the chain rule
 9. Taylor's theorem
 10. local maxima and minima
 11. level surfaces of functions
 12. tangent planes to surfaces in 3-space
 13. Lagrange multipliers
8. Differential Calculus for functions from n -space to m -space
 1. notion of derivative
 2. chain rule
 3. inverse function theorem
 4. implicit function theorem
9. Integral Calculus in several variables
 1. The integral, path and surface integrals

2. Green's theorem in the plane
3. the divergence theorem in 3-space
4. the change of variables formula

Groups and Rings:

Elementary concepts (homomorphism, subgroup, coset, normal subgroup), Lagrange's Theorem, Cauchy's Theorem, commutator subgroup, Sylow theorems, structure of finitely generated Abelian groups, Symmetric, alternating, dihedral, and general linear groups, Commutative rings and ideals (principal, prime, maximal), Integral domains, Euclidean domains, principal ideal domains, polynomial rings, Eisenstein's irreducibility criterion, Chinese remainder theorem.

Number Theory:

Subgroups of the Integers, Greatest Common Divisors, The Euclidean Algorithm, Prime Numbers, The Fundamental Theorem of Arithmetic, The Infinitude of Primes, Congruences, The Chinese Remainder Theorem, The Euler Totient Function, The Theorems of Fermat, Wilson and Euler, Solutions of Polynomial Congruences